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Achievement and attitude effects of a coordinated remedial mathematics laboratory offered concurrently with a college liberal arts mathematics course as compared to a free-standing remedial course.

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ACHIEVEMENT AND ATTITUDE EFFECTS OF A COORDINATED REMEDIAL MATHEMATICS LABORATORY OFFERED CONCURRENTLY WITH A COLLEGE LIBERAL ARTS MATHEMATICS COURSE AS COMPARED TO A FREE-STANDING REMEDIAL COURSE

A Dissertation Presented

by

Frank W. Morgan

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To Nancy

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ABSTRACT

ACHIEVEMENT AND ATTITUDE EFFECTS OF A COORDINATED REMEDIAL MATHEMATICS LABORATORY OFFERED CONCURRENTLY WITH A COLLEGE LIBERAL ARTS MATHEMATICS COURSE AS COMPARED TO A FREE-STANDING REMEDIAL COURSE

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During the last twenty years, a large and growing commitment has been made by colleges and universities to provide basic skills programs in mathematics. The typical program offers separate remedial courses in arithmetic and/or algebra. Deficiencies of the typical program involve a fragmentation of the subject, inattention to purpose and motivation; skills are lost before being applied in subsequent courses; failure rates are high; and negative attitudes towards mathematics are not being addressed.

This study investigated an alternative, dual-purpose program consisting of a one-credit mathematics skills laboratory given concurrently with the beginning college-level, liberal arts mathematics course. It was hypothesized that basic skills students could be remediated while successfully completing the liberal arts course, and that

the dual-purpose treatment would be accompanied by improved attitudes towards mathematics.

The dual-purpose treatment was applied to an experimental group from Castleton State College in Castleton, Vermont. Outcomes for the dual purpose group on arithmetic skills achievement, attitude scale, and completion rate were compared to corresponding outcomes for remedial students taught by a more typical, self-paced method. Outcomes for the dual-purpose students on an achievement test in the liberal arts course, on an attitude scale, and on completion rates were compared to outcomes for two groups of non-remedial students in the same mathematics course.

The results of the experiment indicated that the dual-purpose program was partially successful. The dual-purpose group showed a greater improvement in arithmetic skills and a higher pass rate than the basic skills control group. However, there was no significant difference in attitude among the four groups. Moreover, the presence of the dual-purpose students in the liberal arts mathematics course appeared to have a negative affect on the achievement of the nonremedial students.

Further studies are required to determine the cause of this under-achievement effect in the heterogeneous grouping of remedial and nonremedial students.

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CHAPTER I

INTRODUCTION

Background

Since the late 1960's and the advent of open admissions, many colleges have started basic skills programs in mathematics, reading, and writing. A 1985 survey of a carefully constructed sample of colleges and universities in the United States conducted for the National Center for Educational Statistics (NCES) by Wright & Cahalan, (cited in Schonberger, 1985) indicates that an average of 2.0 remedial mathematics courses per institution were offered.

Typically, these courses include elementary algebra and arithmetic, and sometimes geometry and intermediate algebra. The survey reported that 25 percent of college freshmen took a remedial math class that year, with the percentages varying by type of school from 13 percent in selective and traditional admissions to 30 percent in open admissions colleges.

Quasi-experimental pretest-posttest studies typically find that remedial courses improve basic skills competency for those who complete the course (Schonberger, 1985). However, the NCES survey reports that completion rates ranged from 40 to 50 percent. Evaluation models that consider long range indicators such as success in subsequent mathematics courses are generally discouraging (Schonberger, 1985). Only a few studies have considered how long students retain the skills developed. Rachlin (1981) reports that case studies reveal that certain students identified as successful in a basic algebra course forgot within two weeks how to apply the processes or even that they had studied the topic. Personal mathematics histories of students in developmental courses indicated that 68% had taken similiar courses before. A sizeable percentage didn't know why they were in basic skills mathematics. Frerichs-Eldersveld in a 1981 survey (cited in Schonberger, 1985) found 70 percent expressed negative or neutral attitudes towards mathematics, and over 90 percent said their mathematical ability was average or below.

One dimension of the basic skills problem (Whitesitt, 1982) is the large and growing number of faculty and students involved in such courses. Basic skills mathematics courses may be taught in separate developmental departments or by mathematics departments. Using the 25% figure mentioned above, a university with 3000 entering freshmen would require 20 to 30 sections per year for basic skills mathematics classes, assuming 20 to 30 students per section. This requires the equivalent of 4 to 7 full time instructors depending on normal teaching load. The trend has been questioned, given both the limited availability of funds and the limited effectiveness of remedial courses (Gemignani, 1977). Use of adjunct faculty is common (Schonberger, 1981). While adjunct faculty are usually capable and conscientious, it is difficult to maintain quality control and continuity because of changing personnel. Contact with students outside the classroom is difficult for adjunct faculty. It is not surprising that the basic skills courses are often farmed out to adjunct faculty in light of the low esteem associated with teaching such courses (Wepner, 1986).

Schonberger (1985) reports that the traditional lecture-discussion instructional method predominates in basic skills courses with personalized systems of instruction (PSI) also common, including the use of computer assisted instruction. Criticisms about basic skills instruction typically center around the fragmentation of the subject (Rachlin, 1981), the general inattention to purpose and motivation in the available materials (Pace, 1981), and the inattention to the variety of learning styles, developmental levels, motivational needs, and problem-solving skills of the learner (Upchurch, 1980). The programs seem to work reasonably well for those who only need a review (Whitesitt, 1982), but not for those who have attitude, understanding, and anxiety problems with basic mathematics.

There has been much research focusing on basic skills mathematics education in the last fifteen years. Schonberger (1985) reports that status studies, placement mechanisms, and evaluation models are well developed. There have been numerous studies of instructional technique, as well as student characteristics such as attitude and anxiety. Investigations have typically been one dimensional and nonsequential, and have not resulted in any comprehensive solutions (Schonberger, 1985). Teaching systems, attitudes, motivations, learning styles, thought processes (less frequently), and class management have been studied as one dimensional variables with limited success in identifying solutions to the persisting problems such as high failure rates. The dimensions listed above have not been addressed comprehensively in a context where understandings of meaning and purpose (Pace, 1981), mathematical content, and the nature of mathematics are developed along with basic skills.

An alternative approach to basic skills mathematics is the provision of parallel remediation through the use of scheduled mathematics laboratories for remedial sessions coordinated with a particular nonremedial course, with both under the supervision of the same professor. Although some schools may do this, reports of investigations of this approach do not exist in the mathematics education literature. More typically for college level courses, mathematics laboratories are available on a voluntary basis (Rosamond, 1981), or are not oriented to a specific course.

It seems clear that parallel remediation would not work for some courses. For example, the student who takes a calculus course will need a good grounding in algebra before starting calculus (Whitesitt, 1982). However, the liberal arts mathematics courses typically offered for humanities majors, called Elements in the remainder of this proposal, might offer a context in which parallel remediation makes sense, especially in view of students whose interests and career plans do not involve technical or higher mathematics.

There is a sparsity of research in the mathematics education literature on Elements type courses, in spite of the obvious curriculum and instruction problems associated with this course. The same problems attendent with basic skills are there for Elements, along with a greater diversity of student aptitude, attitude, interest, and need. Typically, this would be the course that would be elected by basic skills students to fulfill a general education mathematics requirement. More work on models for this course are needed, both to provide a context for parallel remediation, and to improve the present Elements course.

Statement of the Problem

Present state: Basic skills programs at the college level typically suffer from:

1. Skills being lost due to disuse before they are applied in subsequent courses or vocational applications.
2. High student anxiety and low self-concept of ability.
3. Little improvement in problem-solving and other higher level skills.
4. Poor attitudes towards the subject.
5. A high failure rate.

Goal state: Basic skills mathematics programs at the college level are so designed and executed that basic arithmetic and algebraic skills are improved _concurrently with_:

1. Immediate skill application.
2. Development of problem-solving and higher level skills.
3. High anxiety being decreased through improved self-concept of mathematics ability.

4. A gain in appreciation of the beauty, power, and utility of mathematics along with its limitations and possible abuses.

5. A completion rate as high as that of other first semester freshmen courses at that college.

Purpose and Rationale

This problem is worthy of intensive study at this time because:

1. The high dropout rate and other shortcomings in college remedial mathematics programs is a national problem.

2. The attempts to find solutions within a separated, strictly remedial, no credit program have not been especially successful.

3. A reasonable alternative has been suggested but not tested. (The CUPM model to be described below).

4. If the experiment described below is successful it will provide administrators, faculty, and students with a satisfying and efficient basic skills option that is no less effective in remediation.

Definitions

1. The Keller Plan or Personalized System of Instruction (PSI) is an instructional system characterized by the following features (Keller, 1968):

(1) self-paced, (2) mastery-based, (3), lectures used for motivation rather than critical sources of information, (4) emphasis on written communication between student and teacher, such as the use of study guides to direct the student into new content, and (5) the use of frequent testing/retesting with proctors at a ratio of 10-1 with students.

2. Problem-solving or inductive approach uses problems or examples to introduce new material in a concrete form, than generalizes later.

3. Traditional or deductive approach presents a general rule or method first, than illustrates it with examples.

4. Computer Assisted Instruction (CAI) refers to teaching systems where the student uses a computer to access information, procedures, drill problems, and tests. This method is often used with PSI.

5. Advanced organizer is an abstract or general idea presented as introductory material and designed to provide a structure for the understanding and retention of more detailed material to follow.

6. Behaviorial objectives are stated clearly in terms of behavior so that the learners can be sure that they have mastered the objectives.

7. Field-dependence-independence is a dimension of

cognitive style that is assessed through perceptual tasks in which performance is influenced by information from the visual environment. The field-dependent style individual is reliant on the contextual information, while the field-independent style individual is able to perform the task without being dominated by the surrounding field.

8. The dynamic principle states that the understanding of a new mathematical concept is an evolutionary process which is enhanced by involving the learner in three ordered stages. The play stage provides the learner with an opportunity to interact informally with materials, situations, or an environment containing "concrete" structures within which the mathematical concept of interest is embedded. The learner begins the process of observing regularities and rules (Reys & Post, 1973). Following the play stage, the learner is involved in a period of structured activity (becoming aware stage) in which the relevant variables with their relations and operations are highlighted to assist the learner in becoming aware of these relationships. In the final stage the concept becomes fully operational (operational stage). This includes the ability to recognize freely the concept and apply it in relevant situations with the same mathematical structure. The now-familiar concepts and

symbols become part of the environment for the next concept to be learned, and the cycle repeats.

9. The multiple embodiment principle affirms that conceptual learning is optimized when the concept is presented in a variety of situations that appear different (color, texture, media used, etc.) but have the same basic structure.

10. The mathematical variability principle states that abstraction of a mathematical concept will be enhanced by taking care to vary the irrelevant mathematical variables while leaving the relevant variables constant.

Outline of the Study

The purpose of this project was the development of an alternative Basic Skills Program in mathematics coordinated with a college general education mathematics course (Elements). This program was called the Dual-purpose Treatment, since it involves the remedial student concurrently in two courses designed to (1) develop basic skills through a laboratory course, and (2) concurrently develop the knowledge and capabilities associated with successful performance in the Elements course.

The remainder of this study will describe (1) the current issues and research in mathematics education that impact the problem, (2) the development of a dual-purpose program to meet the need, (3) the experiment that was used to evaluate the program, (4) the results of the experiment, and (5) conclusions and implications for further research.

CHAPTER II

RELATED RESEARCH AND ITS IMPLICATIONS

Introduction

This chapter contains a review of recent studies involving remedial mathematics programs offered by colleges as well as studies involving general education mathematics courses at the college level. The research results are presented in the form of issues involving commonly compared variables, including problem-solving vs. deductive approaches, directed study vs. teacher exposition, advance organizers, and mathematical attitudes and anxiety. The impact of this research on the design of the dual-purpose model is considered in detail.

How Others Have Addressed the Problem

Typically, experimentation with basic skills improvement at the college level has attempted to isolate variables that cause desired outcomes, within the context of a separate remedial course or program of study. Many of these studies examined effects of instructional modes, comparing a traditional lecture method to problem-solving/inductive approaches, or personalized systems of instruction/CAI/contract approaches, or advanced organizer based instruction, or instruction using peer tutors. Frequently, a second independent variable such as cognitive style was involved, with a focus on the interaction between instructional mode and cognitive style. Cognitive styles considered include field dependence/ field independence, thought processes, and teaching/learning style match. Mathematics anxiety, attitude, and self-concept of mathematics ability have been examined, often in the presence of different instructional treatments or relative to cognitive style. Altogether, these studies have contributed to our understanding of the effects of certain variables or to the interactions among a few variables, and provide some direction about the components that might be built into a more comprehensive program.

Problem Solving vs. Deductive Approaches

Treadway (1983) suggested that a "real problem solving approach" to the acquisition of mathematical and problem-solving skills would be a basis for a more effective general education mathematics course. The problem-solving group showed a greater improvement in attitudes toward mathematics and scores on tests of mathematical skill than the control group, but not significantly. However, students in the problem-solving group showed better problem-solving skills at the end of the semester. Meyer (1982) found improvement in attitude but not achievement as the result of the use of specific problem solving heuristics. Layne (1982) found two problem solving methods equally effective when employed by poorly prepared college students to solve both simple and novel mathematics problems. He found the "translation/computation" method to be better that "Polya's step method" for these same students when solving complex mathematical problems. Payne (1983) found that three problem solving techniques were equally effective in significantly improving problem-solving ability and in retention of learning in a general education mathematics class. These studies show that <u>attention to problem solving is appropriate for poorly prepared students in general education mathematics courses.</u>

However, an inductive problem solving approach may not be equally effective for all student learning styles. Horak (1977) found that an inductive method is better than a deductive method for producing transfer of learning to new situations. She found that it is desirable for field-dependent students to be taught by an inductive method. This is relevant for basic skills, since text materials and traditional instructional procedures typically are based on a deductive approach, and many basic skills students are presumably field dependent. Clute (1984) found that students taught by the discovery method outperformed those taught by the expository method on high level test items. However, students with high mathematics anxiety tended to do better under an expository treatment, while those with low anxiety did better under a discovery method. These studies show the importance of <u>a balanced treatment that uses inductive, problem-solving approaches frequently as well as clear exposition</u>.

<u>Directed Study vs. Teacher Exposition</u>

Several studies, including Chairamonte (1979), King (1979), and McDonald (1983), found no significant difference between traditional instruction and self-paced, modularized, or contract methods. Diem (1982) found a conventional lecture, homework method having higher, but not significantly higher scores than groups using varying amounts of computer tutorial and drill. This may be explained by the different learning styles of students. Wilson (1981) found evidence that field dependent students should be matched with instructor pacing, while field-independent students should be matched with self-paced instructional modes. However, Showalter (1981) found that field-dependent and field-independent students performed equally well irrespective of whether the method was high-support (study guides, short lectures, and study groups) or low-support (same lectures with individual study). These studies suggest that a balance which presents some material via clear teacher exposition and some material via directed study would be fair while promoting adaptibility.

Advance Organizers

Although some reviews of studies involving advance organizers have shown mixed results (Barnes & Clawson, 1975), some recent studies seem useful. Rodman (1982) found that students who studied with the benefit of advance organizers performed significantly better than those using behavorial objectives. This is relevant since the use of behavorial objectives is typical for many basic skill textbooks and personalized systems of instruction. Anderson (1978) found that advanced organizers significantly facilitated the learning of the more difficult concepts. Doyle (1981) used an advance organizer in a remedial college mathematics class to anchor a formal level, mathematical concept which in turn significantly facilitated learning, transfer, and retention. However, logical reasoning level was a significant correlate of ability to understand the advance organizer. Rodman (1982) points out that proper construction of advance organizers is an exacting and demanding task. These studies suggest that the effectiveness of advance organizers would vary with the cognitive style of the learner, perhaps being especially useful for field dependent students. These studies suggest that _advance organizers should be used whenever possible, but not to the exclusion of behavioral objectives._

Mathematical Attitudes and Anxiety

A large number of studies in basic skills mathematics have assessed the relationship of math anxiety and attitude to achievement. Several have found no significant correlation between anxiety and achievement (see Lowe (1982), Gourgey (1982), Freeman (1982). However, Gourgey found that math self concept was significantly correlated with achievement, and Delventhal (1982) found that attitude was significantly correlated with achievement. Crumpton (1977) found that increasing mathematical competence is an effective method of reducing math anxiety, and that non-mathematical treatments may have had little effect upon reduction of math anxiety. This is in line with the findings of Glass (1982) that the combined use of counselling and academic-intervention techniques was not effective in reducing math anxiety, test anxiety, increasing attitude towards mathematics, or increasing performance. Use of peer tutors was found by McKeithan (1982) to result in a significantly higher mean score on an attitude scale. Phuvipadawat (1984) found that students studying within a cooperative goal structure had a significantly lower test anxiety when compared with those studying within a competitive or individualistic goal structure. These studies are highly relevant parts of the basic skills picture, since basic skills students are prone to have high anxiety and bad attitude towards mathematics. These studies indicate that <u>mathematics attitude and anxiety should be addressed through the cultivation of collaborative goal structure and through increasing mathematical competence.</u>

<u>The CUPM Model</u>

The studies described above collectively have provided direction for improving basic skills courses. However, if all the superior methods were somehow integrated into a single teaching system and applied in a strictly remedial course, the fact remains that students must typically postpone taking a college level course until remediation is complete. Many will delay mathematics further, or avoid it entirely unless it is a general education requirement. Thus it is appropriate to explore the feasibility of developing skills while a student is concurrently enrolled in a coordinated general education mathematics course.

The idea of such a course is not new. In their booklet <u>A Course in Basic Mathematics</u> (Boas, 1971), the Committee on the Undergraduate Program in Mathematics (CUPM) described a course aimed to "provide students with enough mathematical literacy for adequate participation in the daily life of our present society." By this they apparently intended something like goals 2 and 4 of the goal state objectives listed above, dealing with the development of problem solving and higher level skills and an appreciation of the beauty, power, and utility of mathematics. The course would provide an adjunct laboratory to remedy the deficiencies in arithmetic that "so many students possess." (Boas, 1971). The laboratory would also provide opportunity for algebra skills development.

The CUPM model was presented with the hope that many different kinds of institutions would find good use for its suggestions. They assert that their proposal is broad and relevant to the actual concerns of students, perhaps more appropriate as a genuine liberal arts course than most of the courses currently taught for this purpose for students of this level. The committee suggested that the spirit of their proposal is more important that the content. In their words, "From the standpoint of general education, the proposed course is a broad one; it can be termed the mathematics of human affairs, and as such should be a reasonable alternative to the usual general education course. Moreover, the students...are likely to be of a pragmatic turn of mind. For them an appreciation of mathematics seems likely to stem from seeing how mathematical ideas illuminate areas in which they have an established interest." (Boas, 1971).

Although the CUPM proposal carries considerable prestige, the mathematics education literature since 1975 did not reveal reports of research comparing this model to the more common approach of separated remedial courses. Perhaps the committee's proposal has been implemented, but the literature does not contain any studies of programs that coordinate the liberal arts course carefully with the laboratory.

Dienes' Model for Mathematics Learning

The spirit of the CUPM recommendations as well as the research issues described above can be integrated nicely using the learning theory of Zoltan P. Dienes (1960). Building on the cognitive learning theory of Piaget, Dienes views mathematics as the study of actual structural relationships involving numerical and spatial concepts together with their applications to problems in the real world. He views mathematics learning as "the apprehension of such relationships together with their symbolization, and the aquisition of the ability to apply the resulting concepts to real situations occurring in the world." (Reys and Post, 1973). The relationship of the mathematical structure to the real world structure is that of the abstract to the concrete. Here abstract means simply void of unneccessary detail.

The primary component of Dienes' learning theory, the **dynamic principle**, flows out of his structural view of mathematics and his linking of concrete and abstract structures. The dynamic principle states that the understanding of a new mathematical concept is an evolutionary process which is enhanced by involving the learner in three ordered stages. The play stage provides the learner with an opportunity to interact informally with materials, situations, or an environment containing "concrete" structures within which the mathematical concept of interest is embedded. The learner begins the process of observing regularities and rules. Dienes believes that the mathematical concepts should be constructed in a holistic, intuitive way as a consequence of individual experience. Later, attention can be directed toward analysis of what has been constructed, but "it is not possible to analyze what is not there." (Reys &

Post, 1973).

Following the play stage, the learner is involved in a period of structured activity (becoming aware stage) in which the relevant variables with their relations and operations are highlighted to assist the learner in becoming aware of these relationships. This might be done through questioning which leads to a discovery of regularities in situations and subsequent experimentation with new found rules or constraints. This stage, involving comparison of several situations having identical structure or rules, can appropriately be described as a search for isomorphisms. Dienes (1960) formalizes this process in his **multiple embodiment principle**, which affirms that the same conceptual structure should be in a variety of perceptual situations, such as different color, texture, media used, etc. This promotes abstraction, the ability to perceive a concept in various concrete embodiments. Dienes also cautions that abstraction will be enhanced by taking care to vary as many irrelevant mathematical variables as possible while leaving the relevant variables constant. For example, triangular shapes should be presented in a variety of angle measure or side length configurations. This idea is called the **mathematical variability principle**. (Reys and Post, 1971).

In the final stage the concept becomes fully operational (operational stage). This includes the ability to recognize freely the concept and apply it in relevant situations with the same mathematical structure. In this stage the formal definitions are established and the conventional notation is mastered. This stage includes statements and proofs of theorems, and derivations of algorithms and practice in their use. The now-familiar concepts and symbols become part of the environment for the next concept to be learned, and the cycle repeats. Dienes makes it clear that concrete materials must be used for young children, but that mental games are appropriate for the play stage of older children and adults (Dienes, 1960).

Implications of the Model

This learning model suggests many insights into both the normal and pathological products of mathematics education. This study considers the impact of the model as it relates to the outcomes of the studies described above, considering in turn issues involving problem solving, the use of clear exposition, the use of directed study, the use of advance organizers and behaviorial objectives, and the problems with mathematics anxiety and poor attitude.

An emphasis on problem solving dovetails nicely with

Dienes model. Dienes would suggest that the play stage, and the becoming aware stage have been typically shortchanged in the rush to get to the formal definitions, notation, theorems, and algorithms. This emphasis may leave the student with the view that mathematics is a collection of unrelated techniques, and the decision as to which technique to use in a given situation is learned by rote or through a process of association such as the use of key words to suggest a particular operation. The problem with this "bag of tricks" approach arises when the student is faced with a situation in which he doesn't have a ready response pattern. In that situation, it is not likely that he will be able to specify the mathematical structure of the embodiment, not having been trained to do so. Again, according to Dienes' model, his inability is a result of an over-emphasis on the operational stage in the dymanic process. Attention to the dynamic principle of Dienes may be a way to compensate for the remedial student's (as well as the Elements student's) dislike of word problems and problems in general.

Both clear exposition and directed study can be used naturally within Dienes' model. Directed study is primarily used in the play stage. The environment is selected by the teacher in such a way that a particular structure is embodied in the situation. After the student gains familiarity with the situation and begins to observe some of the regularities, his attention is directed toward the structure, including relationships and modes of operation within that environment. Up to this point, exposition is kept to a minimum, and creativity is encouraged. Once the student is aware of the structure of interest, clear exposition is the most efficient way to present the common codes that are used to talk about the concept, represent it, establish its validity in general (proof), and manipulate it in algorithms. This is especially true since the codes are often arbitrary in the sense that other codes could be invented for the same concept. But our creative energy is saved for conceptual work rather than for inventing redundant codes. Directed study would than be used in applications of modelling with the newly learned structure. The cycle repeats through the introduction of new structural elements into the environment.

An advance organizer, as proposed by David Ausubel (1963, cited in Joyce & Weil, 1972), is a general idea presented as introductory material that is more abstract and precedes the learning task. The advance organizer should "provide ideational scaffolding for the stable incorporation and retention of the more detailed and differentiated material that follows." Dienes' model harmonizes well with Ausubel's use of organizers. The idea that every mathematical concept is structural and has various embodiments enhances the **stablization** of the new idea as required by Ausubel. Also, the whole dynamic process could be termed **guided discovery** as promoted by Ausubel. Global concepts, such as **mathematical systems** and **mathematical modeling** would be developed through the dynamic process, and then be used as advance organizers for the content developed throughout the course. For example, the concept of a mathematical system is used as an advance organizer in the development of set theory, mathematical logic, number and numeration systems, all of which are introduced dynamically as particular systems, enlightened by and in turn giving light to the theme of **mathematical system**. Similiarly, **mathematical modeling** is developed throughout the course, for example in counting problems, clock arithmetic, descriptive statistics, and equilibrium problems.

Behavioral objectives are ideal in pointing out the exact processes and for developing the precision involved in the use of notation and common code. This is a natural part of the third stage when results are formalized, algorithms are established, proofs are made, and additional applications are obtained.

The final implication of Dienes' model to this study is its relationship with mathematical anxiety and poor attitudes toward mathematics. Just as problem solving skills have been short-circuited by omission of a play stage and a becoming aware stage in traditional methods of mathematics teaching which emphasize drill, so also normal attitudes and emotional states have been a casualty for many. In the light of Dienes model, anxiety comes from a failure to build the conceptual structure before the formalization stage. Symbols and algorithms used by rote without conceptual understanding and structural embodiments are an invitation for anxiety and poor attitude. One is not anxious about what he understands well. Conversely, he feels anxious about being called on to perform in areas where he lacks understanding, especially when others know the code and tell him it is easy. This effect is magnified by the differences in developmental level typically found among the students in classrooms at all levels. Lack of understanding and inability to apply is self-reinforcing in a destructive way. Breaking down years of reinforcement of these attitudes will require patience, encouragement, and attention to the dynamic principle.

Summary

Remedial mathematics at the college level has mushroomed since the early seventies, and has been the subject of much research involving the effect of various instructional treatments on achievement and attitude in light of various learner characteristics. The research clearly indicates a variety of effects and inter-relationships among these variables which points to a need for (1) variety in teaching style, approach, and form of instruction, (2) use of advance organizers and behaviorial objectives, and (3) attention to mathematical anxiety through an emphasis on cooperation and increasing competence.

The impact of this research on the present study involves the need to incorporate these variables into a dual-purpose program, similiar to the **Basic Mathematics** course suggested by the CUPM. This was done in the context of a learning theory model that gave coherence to the program in its remedial and liberal arts mathematics components with the intent to enhance achievement, attitude, and persistence of the basic skills college student.

CHAPTER III

RESEARCH METHODS

Introduction

The purpose of this study was to investigate the achievement, completion rate, and attitude effects of a dual-purpose, basic skills program consisting of a coordinated remedial mathematics laboratory offered concurrently with a college liberal arts mathematics course. The dual-purpose program was compared to a more typical three semester-hour remedial course in arithmetic. The study also investigated the effects of heterogeneous grouping as compared to homogeneous grouping on achievement in a liberal arts (Elements) mathematics course. The present chapter will describe the population and sample, treatments and controls, hypotheses, instrumentation, statistical models and analysis, procedures and schedules, learning theory applications, and mathematical content of the various treatments used in conducting the study.

The Population and Sample

The experiment involved two populations of college undergraduates, with an experimental and control group from each. The test site was Castleton State College in Vermont. Castleton State College is a comprehensive institution with an enrollment of over 1700 students offering a wide range of undergraduate programs in the sciences, the arts, the humanities, and career and professional fields. A strong liberal arts core curriculum supports all degree programs. Included in the core is a six-credit mathematics requirement. Another part of the core curriculum is the Academic Skills Program, that requires new students to sit for tests in writing, reading, and mathematics prior to their first registration. Those who fail are required to enroll in and pass basic skills courses.

Population 1: This population consists of beginning college freshman students who require basic skills (remedial) training in arithmetic skills and algebra skills.

Basic Skills Experimental Group: a sample of 27 Castleton State College freshmen selected randomly from among the 55 entering freshmen who failed both the Arithmetic Skills and Algebra Skills tests. These students were to be enrolled concurrently in both a basic skills laboratory and the Elements course, and so are called the Dual-purpose Treatment Group, or more briefly, the Dual-purpose Group.

Basic Skills Control Group: a sample of 28 Castleton State College Freshmen consisting of those basic skills students not selected for the Dual-purpose Group.

Population 2: This population consists of college undergraduates who do not require basic skills training in mathematics, and whose curriculum requirements include a liberal arts course in mathematics similiar to the Elements course described above. These students typically are majors in curriculums that do not require advanced algebra/trigonometry or calculus.

<u>Experimental (Heterogeneous) Elements Group</u>: a sample of 33 Castleton State College undergraduates selected through the normal registration process to enroll in a heterogeneous section of the Elements course -- that is, in a section with members of the Dual-purpose Group. This sample consisted of 25 entering freshmen who had passed the mathematics basic skills tests and 8 returning or transfer students who either did not require or had already completed mathematics skill remediation.

<u>Control (Homogeneous) Elements Group</u>: a sample of 28 Castleton State College undergraduates selected through the normal registration process to register for a homogeneous Elements section; that is, a section not containing any members of the Dual-purpose Group. This sample consisted of 11 entering freshmen who had passed the mathematics basic skills tests and 17 returning or

transfer students who either did not require or had already completed mathematics skill remediation.

Treatments

The Basic Skills Experimental Group members were enrolled in a 1-hour (no graduation credit) basic skills laboratory that was designed to provide remedial skill development in arithmetic and elementary algebra. The content and teaching system of this course are described later in this chapter. Appendix A includes some course materials for the basic skills laboratory. The members of this group were also concurrently enrolled in one of the heterogeneous sections of the Elements course (three graduation credits). This course was designed to be a typical, replicable liberal arts course in mathematics for students whose career plans and program needs did not require a more technical mathematics course. This course was designed along the lines of the CUPM course described in Chapter 2. A discussion of the course content and materials is included later in this chapter, and course materials are included in Appendix B.

The Basic Skills Control Group was enrolled in a 3-hour (no graduation credit) Basic Skills course in arithmetic. This course uses a typical self-paced, Keller-plan instructional model. A directed-study type workbook/textbook was the primary source of learning material. The content and teaching system of the course are described later in this chapter, and course materials are included in Appendix C.

The Heterogeneous Elements Group members were enrolled in a heterogeneous section of the Elements course along with members of the Dual-purpose Group, at a ratio of about 11:9. Other than the presence of the Dual-purpose students in the same section, the treatment was designed to be that of a typical, replicable, liberal arts course in mathematics designed along the lines described above for the CUPM model. Additional information about this course may be found in Appendix B.

The Homogeneous Elements Group members were enrolled in a homogeneous section of the Elements course that did not contain any remedial students. Otherwise, the treatment was as similiar to the Heterogeneous Elements Group as possible.

Controls

The two Basic Skills Groups were selected using a table of random numbers. Because of course conflicts and the requirement that basic skills remediation be taken the first semester on campus, three students were allowed to switch groups. The mathematical content, approach, and instructor were different for the two basic skills groups. The Control Group recieved three hours of instruction per week (no graduation credit) in arithmetic skills, while the remedial work for the Dual-purpose Group involved only one hour per week (no graduation credit) of remedial work, which included some study skills and problem solving activities as well as arithmetic. It was thought that the use of arithmetic skills in the Elements course would compensate for the smaller amount of time used on explicit arithetic skills instruction, as per the CUPM recommendations. Thus the content and approach were intended to be different due to treatment. The difference due to the instructor was thought to be negligible due to the teaching system in the control group being a Keller-plan type course. Both groups made use of paid tutors provided by the college learning center. The content of all screening tests were the same for the two treatments. The pretest and posttest were identical for the two groups.

The mathematical content and teaching system used in the Elements course were designed to be identical for the dual-purpose group, the heterogeneous group, and the homogeneous group. To this end, course policy, pacing, syllabus, study sheets, lesson plans, assignments, and instructor were selected to be identical. All groups had equal access to paid tutors supplied by the learning center. Although there were other sections of Homogeneous Elements being taught, it was decided that the difference in instructor, teaching style, and content emphasis would make the groups less comparable. Two sections of homogeneous Elements taught by a colleague of the investigator were used as a pilot group for the Elements achievement test.

Table 1: Summary of Groups and Treatments

Group	Treatment
Basic Skills Control	3-hour Remedial Arithmetic
Dual-purpose	1-hour math lab & 3-credit Elements
Heterogeneous Elements	3-credit Elements course only (with Dual-purpose remedial students)
Homogeneous Elements	3-credit Elements course only (without remedial students)

Table 2: Comparisons

Dependent Variable	Groups Compared
Arithmetic Skills	Basic Skills Control Dual-purpose
Elements Achievement	Dual-purpose Heterogeneous Elements Homogeneous Elements
Attitude	Basic Skills Control Dual-purpose Heterogeneous Elements Homogeneous Elements
Basic Skills Pass Rates	Basic Skills Control Dual-purpose
Elements Course Pass Rates	Dual-purpose Heterogeneous & Homogeneous

Experimental Hypotheses

1. The mean pretest to posttest gain in arithmetic skills for the Dual-purpose Group would be greater than or equal to that of the Basic Skills Control Group. It was expected that the smaller amount of time devoted to remediation would be compensated for by the attention to purpose, problem-solving, and immediate application of basic skills in the Elements course.

2. The Heterogeneous Elements Group mean score on the Elements achievement test would not differ from the mean score of the Homogeneous Elements group, but would be greater than the mean score of the Dual-purpose Group. It was expected that the inclusion of the Dual-purpose

students in an Elements section would not decrease the achievement of the Elements students in that section, since all sections would cover the same content. This was due to the sequencing of course material that made it unneccessary to cover basic skills during the Elements classes, since the Dual-purpose students had covered the remedial material recently in their separate but coordinated laboratory. However, it was expected that the Dual-purpose students would remain somewhat behind their non-remedial counterparts in overall mathematical achievement.

3. The mean score on the attitude scale for the Dual-purpose Group would be higher than the mean score on the attitude scale for the Basic Skills Control Group, but would not differ from the mean score of either of the two Elements groups. Although the Basic Skills Control Group is expected to show attitude improvement generated by successfully completing a self-paced course, it was expected that the Dual-purpose Group attitudes would be further enhanced by their math skills laboratory and their involvement in the Elements course with its emphasis on the power and utility of mathematics. Moreover, it was expected that the dual purpose treatment would compensate for any initial attitude deficiencies of the Dual-purpose Group as compared to the Elements

groups.

4. The Dual-purpose Group would have a higher percentage of students pass the skills course than the Basic Skills Control Group. Furthermore, the percentage of students in the Dual-purpose Group who pass the Elements course would not differ from the percentage of students in the Elements groups who pass the Elements course. It was expected that motivation and persistence would be enhanced for the dual-purpose group in comparison to the Basic Skills Control Group by the attention to purpose in the elements course and the fact that the students would be concurrently in a course offering credit. Also, it was expected that the dual-purpose program would be effective in developing the capability needed to complete the Elements Course successfully.

Instrumentation

The Arithmetic Skills Test, Forms A and B, of the "Descriptive Tests of Mathematics Skills" published by the College Entrance Examination Board, was used to measure arithmetic skills. These tests were constructed by Educational Testing Service of Princeton, New Jersey. This test is presently used by the Vermont State College System to place entering freshmen and to measure skills after remediation. Thus, the use of these tests did not require additional expense or inconvenience to the student. Content validity is assumed to be high based on the rigor of the test development process. The ETS staff was assisted by an advisary committee of ten leaders in mathematics education in the process of developing the specifications of the test series. Forty-six mathematics teachers, junior high school through college, were engaged to write the items. The nearly 2000 items were reviewed and edited by the ETS staff, and finally selected items were reviewed by four mathematics teachers. Tryout and revision process resulted in tests that were deemed appropriate in difficulty and discriminating power. Internal consistency as measured by Kuder-Richardson Formula 20 varied for the norm groups from .86 to .91. Equating of the alternate forms was accomplished through special testing programs.

The Aiken Mathematics Attitude Scale (Aiken, 1979) was used to measure attitude towards mathematics. The Mathematics Attitude Scale is a Likert-type questionaire consisting of twenty-four statements about mathematics to which the respondent indicates how strongly he/she agrees or disagrees. Aiken found that the internal consistency as measured by coefficient alpha ranged from .81 to .91. He found test-retest reliability coefficients about 0.9. In a validity study involving college students (Gadzella, 1985), it was found that four factors accounted for 59.9% of the variance, with 16 loading on the first factor and accounting for 40.8% of the total variance. The scale was viewed as more unidimensional than multidimensional for the group examined. It seems desirable to use the results of the present experiment to assess construct validity for the present population. A copy of Aiken's Scale is available in Appendix E.

An Elements achievement test was constructed by the investigator to measure achievement in the Elements course. Details about the construction of the Elements test are available in the Appendix D. The Elements test was based on objectives developed by the investigator. The test was reviewed for content validity by five college teachers of mathematics, and was reviewed for technical quality by an expert in Educational Testing. The test was piloted by a colleague of the investigator in a sample of 45 Castleton undergraduate Elements students similiar in composition to the Homogeneous Elements Group described above. It was found that internal consistency as measured by KR 20 was 0.79. A copy of the Elements test may be found in Appendix D. Test pilot results may be found in Appendix F.

Statistical Models and Analysis

1. Arithmetic achievement effects were measured using a pretest-posttest control group design. Statistical analysis involved a two-tailed t-test on the pretest to posttest gain scores.

2. Attitude effects were investigated using a four-group, completely randomized design. Statistical analysis involved a one-way analysis of variance. Scheffe's test was used to explore differences.

3. Completion rate effects were investigated using a randomized, two-group design. Statistical analysis involved a one-tailed z-test assuming that the two groups represented binomial distributions that were approximately normal.

4. Elements achievement effects were investigated using a three group, completely randomized design. Statistical analysis involved a one-way analysis of variance. Scheffe's test was used to analyze the various paired differences.

Procedures and Schedule

The investigation was carried out during the period June 6 through December 18, 1987. Table 3 below gives the schedule of events.

Table 3: Schedule of Events:

Event	Dates
Freshman Basic Skills Testing Following the administration and scoring of the Arithmetic Skills Test, students who scored less than 26 correct out of 35 were randomly assigned to one of the Basic Skills Groups. Those selected for the Dual-purpose group were also assigned to a Heterogeneous section of Elements.	June 6 & 28, August 2
Registration of Freshmen and trans-students occurred. Nonremedial students registered into Heterogeneous and Homogeneous Elements sections through the normal registration process.	June 7 & 29, August 3
Instruction begins for all treatments.	August 31
Arithmetic posttesting and attitude scale forms completion for the Basic Skills Control Group occurs as students finish their work in the self-paced course.	September 21 - December 14
Elements Achievement Pilot Testing occurs at the end of a four week instructional unit based on the test objectives and taught by a colleague of the investigator.	November 4
Arithmetic posttesting for the Dual-purpose Group occurs at the conclusion of Arithmetic Laboratory instruction. The final two weeks of the semester was used for algebra skills instruction.	November 18 & 19
Attitude scale forms are completed by the Dual-purpose group and by the Homogeneous and Heterogeneous Elements Groups.	November 30 & December 1

Elements Achievements Test is administered to the Dual-purpose Group and to the Homogeneous and Heterogeneous Elements Groups at the conclusion of 11 class hours of instruction on the test objectives.	December 2-4
All treatments completed and pass rates determined.	December 18.

The Dual-purpose Model

The purpose of this study was to design and test a program that would accomplish two purposes simultaneously. First, the program would accomplish the goals of skill remediation through a mathematics skills laboratory, and second, the program would allow the student to complete a college general education mathematics course (Elements). The dual-purpose program was an application of the CUPM model, supplemented with principles gleaned from the research described above, and with elements of mathematics learning theory.

The desired result of the study was a working model for a replicable college-level general education mathematics course with a coordinated basic skills laboratory that would accomplish the goal of skill remediation. The Elements course provided the context in which the nature of mathematics was explored and in which appreciation for mathematics was developed. A parallel one hour laboratory in which the students met weekly for problem solving and skill development provided for the compensatory needs of the basic skills student. This laboratory also was a support group for these students. In view of the lack of success sometimes experienced by basic skills students with subsequent mathematics courses (Eisenberg, 1981), the dual purpose course was designed to give these students the type of help they needed.

Application of Dienes Model

As a general application of the model, the Elements course material was presented using Dienes' dynamic, multiple embodiment, and mathematical variability principles through guided discovery and clear exposition. The first unit presented dynamically the themes of **mathematical modeling** and **mathematical systems** that function as advance organizers for the entire course. Each unit utilizes these themes. Class time was used primarily for "becoming aware" stage activities, and for exposition of common code, formal definitions and algorithms. Cycling into a new play stage was accomplished primarily by study sheets that were to be worked through at home. This provided the time required to build the conceptual structures postulated by Dienes. Homework also included "formal stage" activities. Further elaboration of procedures for the Elements course are available in Appendix B.

The laboratory course was also conducted in a similiar manner, with laboratory activities always assigned as take-home exercises, followed by class discussion and work on exercises, algorithms, and code. Information about the laboratory course is available in Appendix A.

The Basic Skills Control Group were enrolled in a self-paced arithmetic skills course patterned after the Keller-plan teaching system. This course is designed to be self-reinforcing through the successful passing of unit tests. The student moves through the content at his/her own pace. Only behaviorial objectives are used, and a mastery approach is used with student tutors in the classroom. There is no use of advance organizers, and no special instruction in problem-solving other than the typical occasional section on word problems. Directed study is the entire form of presentation, with no exposition or large group work. The Dienes model is used in the textbook through an inductive approach, but a deductive approach is also used frequently. However the dynamic principle is used if it is argued that the math text is the "concrete" material used in the play stage by the student at this level. More information about this course is included in Appendix C.

Mathematical Content

The content of the Elements course follows the general design of the CUPM course (Boas, 1971) as modified and presented in the text *Modern Mathematics: An Elementary Approach* (Wheeler, 1984). This text was chosen because it is close to the CUPM recommendation and because it is presently being used in the Elements course at Castleton. The course could be briefly described as a survey of mathematics. The course content includes topics on the nature of mathematics, set theory and logic, the number and numeration systems, number theory and modular arithmetic, descriptive statistics, and analytic geometry. A complete description of the course, including course policy guide, syllabus, and study sheets, is available in Appendix B. While the material for the entire course impacts attitudes and pass rates, it was decided to limit the achievement test to material that would be new to all the students, and material that would make use of the basic skills material covered by the dual-purpose group, and presumably already familiar to the Elements Groups. The content selected included non-decimal numeration systems, modular arithmetic, and descriptive statistics. The objectives were selected from the text material at a level similiar to, and in some cases higher than, the text exercises in order to include higher level cognitive tasks in the test objectives. A description of the test construction and test objectives is included in Appendix D.

The Laboratory Course reading and exercise material makes use of the first seven chapters of the textbook Mathematics: an Exploratory Approach by Robert Stein (1975). This book was chosen because the content was appropriate and the presentation was ideal for the Dienes' learning model. Along with the text material, some class time was used each meeting to review basic arithmetic skills. Methods and examples were similiar to those of the textbook Arithmetic: A Text/workbook by Miller and Salzman (1981), the text for the Basic Skills Control Group. The content and testing of the Laboratory course are described in Appendix A.

The Basic Skills Control Group members were enrolled in the course called Essential Mathematics that could be best described as a self-paced course in arithmetic skills. The content of the course included the operations on whole numbers, fractions, decimals, ratios and percents. The text for the course was Arithmetic: Text/workbook by Miller and Salzman (1981). The course is further described in the Appendix C.

CHAPTER IV

RESULTS OF THE STUDY

Introduction

The purpose of this study was to investigate the achievement, completion rate, and attitude effects of a coordinated remedial mathematics laboratory offered concurrently with a college liberal arts mathematics course as compared to a more typical three semester-hour remedial course in arithmetic. An experimental dual-purpose program was developed following the general outlines of the CUPM recommendation for Basic Mathematics. The dual-purpose program consisted of a one-credit mathematics laboratory offered concurrently with enrollment in the beginning college-level liberal arts Elements course. The one-credit laboratory was designed to provide arithmetic skills remediation. The concurrent enrollment in Elements would enable these students to commence their core mathematics requirement immediately rather than postponing it until after remediation.

The effectiveness of the dual-purpose treatment was investigated through the following comparisons: (1) the gain in arithmetic skills for this group was compared with the gain of a control basic skills group taught by a more traditional self-paced, three hour-per-week remedial course in arithmetic; (2) the achievement of Elements course objectives by the dual-purpose group was compared to the achievement of their skill-proficient Elements classmates, and to the achievement of a group of skill-proficient students in a homogeneous section of the same course, i.e., in a section <u>not</u> containing any remedial students; (3) all of the groups were compared on attitude towards mathematics; and (4) all groups were compared on pass-rates for their various courses.

The following hypotheses were investigated:

1. <u>The mean pretest to posttest gain in arithmetic skills for the Dual-purpose Group would be greater than or equal to that of the Basic Skills Control Group.</u>

2. <u>The Heterogeneous Elements Group mean score on the Elements achievement test would not differ from the mean score of the Homogeneous Elements group, but would be greater than the mean score of the Dual-purpose Group.</u>

3. <u>The mean score on the attitude scale for the Dual-purpose Group would be higher than the mean score on the attitude scale for the Basic Skills Control Group, but would not differ from the mean score of either of the two Elements groups.</u>

4. <u>The Dual-purpose Group would have a higher percentage of students pass the skills course than the Basic Skills Control Group. Furthermore, the percentage of students in the Dual-purpose Group who pass the</u>

Elements course would not differ from the percentage of students in the Elements groups who pass the Elements course.

This section summarizes the results of this study.

Arithmetic Achievement Effects

Subjects in the Dual-purpose Group and the Basic Skills Control Group were pretested using form A of the Arithmetic Skills Test during the summer registration testing session. Upon completion of the required course work, they were posttested using Form B of the Arithmetic Skills Test. It was hypothesized that the Dual-purpose Group mean increase from pretest to posttest score on the Arithmetic Skills Test would be equal to or greater than that of the Control Basic Skills Group.

Twenty-six students from the Dual-purpose Group and 23 students from the control group took the posttest. The difference between posttest and pretest scores provided the data for testing the hypothesis. It was found that there was no difference in variance between the two groups ($F = 1.28$, critical $F_{.05} = 2.01$). It was found that the mean score was greater for the Dual-purpose Group than for the Control Group ($t = 2.59$, $p = 0.0064$).

The arithmetic test results are summarized in Table 4, below.

Table 4: Summary of Means and Standard Deviations (Posttest-Pretest)

Group	Sample Size	Mean	Standard Deviation
Dual-purpose	26	7.88	5.48
Control	23	5.48	3.03

The hypothesis that the dual-purpose program would be at least as effective as the traditional program in arithmetic skill remediation was supported. In fact, the Dual-purpose Group showed a significantly greater mean pretest to posttest gain than the Control Group. Though the Control Group received more actual instruction in arithmetic skills content, this was offset by the fact that the Dual-purpose Group received a total of four to five hours of instruction in mathematics per week as compared to the Control Group which received three hours per week. Much of this additional time was spent in material that made use of arithmetic skills.

Elements Achievement Effects

An Elements Achievement Test was administered to students in three groups: the Dual-purpose Group, the Heterogeneous Elements Group, and the Homogeneous Elements Group. All groups were taught by the investigator using a single syllabus and identical lesson plans. It was hypothesized that the mean score for the Dual-purpose Group would be less than the mean score for the Elements Groups and that there would be no difference between the mean scores of the Elements Groups. An analysis of variance was conducted using a completely randomized design. It was found that there was a significant difference among the means ($F = 4.37$, $p = 0.0058$). Results are summarized in Table 5.

Table 5: Analysis of Variance

Source	SS	DF	MS	F	P
Among	265.31	2	132.65	6.72	0.0020
Within	1618.74	82	19.74		
Total	1884.05	84			

Using Scheffe's test, it was found that the mean score for the Dual-purpose Group was less than the mean score for the Homogeneous Elements Group ($p = 0.006$), but did not differ from the Heterogeneous Elements Groups. The Homogeneous Group mean score was greater than the mean for the Heterogeneous Group ($p = 0.012$). Results are summarized in Table 6.

Table 6: Summary of Means for Elements Test

Group	Sample Size	Mean Score	Standard Dev.
Dual-purpose	24	15.33	3.95
Heterogeneous	33	15.91	4.49
Homogeneous	28	19.39	4.77

The hypothesis that the Elements achievement test mean score of the Dual-purpose Group would be less than that of the Heterogeneous Elements Group was not supported. However, this appears to indicate underachievement of the Heterogeneous Elements Group rather than the effectiveness of the Dual-purpose treatment, since the mean score of the Homogeneous Elements Group was significantly higher than that of the Heterogeneous Elements Group. It seems that the presence of the remedial students in a section with nonremedial students did adversely affect achievement of those non-remedial students.

Because of the importance of this finding, additional analysis was carried out to investigate other possible causes of variation between the Homogeneous Group and the Heterogeneous Group.

The proportion of entering freshmen in each group was analyzed. Using a pooled estimater for the difference in binomial parameters, it was found that the Heterogeneous Group had a significantly higher proportion of freshmen than the Homogeneous Group ($z = 2.93$, $P < 0.004$). Results are summarized in Table 7.

Table 7: Proportion of Freshmen in Nonremedial Elements Groups

	Heterogeneous	Homogeneous
Freshmen	25	11
Total	33	28
Proportion	.76	.39

In order to eliminate the variation due to the larger proportion of upperclassmen in the Homogeneous Group, it was decided to compare the results for <u>entering freshmen</u> only in the two groups. Comparing the Arithmetic scores, it was found that the Homogeneous Group mean arithmetic pretest score was significantly higher than the mean score for the Heterogeneous Group. ($t = 1.82$, $p < 0.07$). Results are summarized in Table 8.

Table 8: Mean Scores on Arithmetic Pretest for Nonremedial Groups

Group	Mean	Standard Deviation
Homogen	32.45	2.62
Hetero	30.46	3.17

The question was asked whether this difference in basic skills capability could have accounted for some of the variation between the two groups in Elements Achievement. To control for this source of variation, an analysis of covariance was conducted for the first semester freshmen in the Homogeneous and Heterogeneous

Elements Groups using the Elements Achievement Test score as dependent variable and Arithmetic Skills Pretest score as covariate. It was found that there was a significant difference in adjusted means (F = 10.48, P = 0.0027). The results are summarized in Tables 9 and 10.

Table 9: Analysis of Covariance

Source	DF	ADJ SS	MS	F	P
Between	1	205.07	205.07	10.48	0.0027
Within	34	665.17	19.56		
Total	35	870.25			

Table 10: Summary of Means and Standard Deviations for Nonremedial Freshmen

Group	N	Cov Mean	Cov SD	Dep Mean	Dep SD	Adj Mean
Homogen	11	32.45	2.62	21.27	3.80	20.90
Hetero	26	30.46	3.17	15.35	4.66	15.50

Results of the analysis of covariance confirmed that there was a significant difference in achievement between the two groups of freshmen, and the presence of the remedial students in the section with the Heterogeneous Group Freshmen apparently did adversely affect achievement.

The next question to be explored was the outcomes for the non-freshmen among the nonremedial groups. An analysis of variance was conducted for the following groups of nonremedial students:

1. The freshmen from the homogeneous group (HOMF).
2. The upperclassmen from the homogeneous group (HOMU).
3. The freshmen from the heterogeneous group (HETF).
4. The upperclassmen in the heterogeneous group (HETU).

The means were ordered as follows: HOMF > HOMU > HETU > HETF. The only significant difference in means was between HOMF and HETF ($p = 0.008$). Results are summarized in Tables 11 and 12.

Table 11: Analysis of Variance Among Nonremedial Groups.

Source	SS	DF	MS	F	P
Among	284.83	3	94.94	4.56	0.0061
Within	1206.41	58	20.80		
Total	1491.24	61			

Table 12: Summary of Means for Nonremedial Groups.

Group	Sample Size	Mean Score	Standard Deviation
HOM. FR.	11	21.27	3.80
HOM. NON-FR.	17	18.18	5.04
HET. FR.	26	15.35	4.66
HET. NON-FR.	8	17.38	4.00

The effect of the treatment seems to be more pronounced for freshmen than upper-classmen. This seems to indicate two factors in the difference between the Homogeneous Group and the Heterogeneous group. (1) The presence of remedial students in the Heterogeneous group apparantly had a negative effect on the other freshmen in that group. (2) The presence of the upper-classmen may have had a positive effect on the freshmen in the Homogeneous group.

A negative effect due to the presence of the remedial students may be explained as follows:

1) The presence of the remedial students in the Elements course had an indirect effect on instruction. For example, the instructor may have unwittingly transmitted lower expectations, leading to lower effort among the nonremedial students in that course.

2) The presence of the remedial students directly affected instruction due to the type and content of questions asked by remedial students and subsequent class discussion resulting in less or different content covered in class, or by attention to tasks at a lower cognitive level.

3) The nonremedial students in the class were aware of the fact that some of their classmates were remedial students, leading them to believe that the course would be easy and require little effort.

It is the opinion of the investigator that all of these factors may well have contributed to the differences in achievement.

Attitude Effects

The Aiken Attitude Toward Mathematics Scale was administered to the Dual-purpose Group, the Basic Skills (Arithmetic) Control Group, the Heterogeneous Elements Group, and the Homogeneous Elements Group. It was hypothesized that the Arithmetic Control Group mean score would be less than the mean scores of the other groups. A multivariate analysis of variance was conducted. It was found that there were no significant differences among the group means ($F = 2.31$, $p = .08$). Results are summarized in Tables 13 and 14.

Table 13: Summary of Means on Attitude Scale

Group	Sample Size	Mean	Standard Dev.
Dual-purpose	24	41.6	14.4
Arithmetic	22	44.5	10.7
Het. Elements	30	48.8	14.1
Hom. Elements	28	51.6	18.5

Table 14: Analysis of Variance (Attitude Scale)

Source	Df	SS	MS	F	P
Among	3	1535.28	511.76	2.31	0.0808
Within	100	22149.63	211.5		
Total	103	23684.92			

The hypothesis that the experimental program would result in improved attitudes towards mathematics was <u>not</u> supported. There were no significant differences among group means on the mathematics attitude scale. In retrospect, this conclusion is not surprising. The gain in awareness of the meaning and importance of mathematics that was expected as a consequence of attendence in the math skills laboratory and the Elements Course was most likely offset by the additional difficulties experienced by these dual-purpose students as compared to those in the control group who only had to contend with an arithmetic course that for many was merely a refresher. Also, attitudes towards mathematics formed over a period of years are not easily changed. Additional study of this problem will be required. It may be possible to use factor analysis to determine if Aiken's Attitude Test is multi-dimensional and able to differentiate between such attitude factors as 1) enjoyment of mathematics, 2) importance of mathematics, 3) motivation in mathematics, and 4) fear of mathematics. Aiken (Aiken, 1979) found that three factors are being measured, which he called 1) enjoyment or interest, 2) perceived importance or value, and 3) freedom from fear or anxiety. Gadzella and Davenport (1985) found evidence of four factors accounting for 59.5% of the total variance. However, 16 of the 24 items loaded on the first factor which accounted for 40.8% of the total variance. They concluded that the scale was more unidimensional than multidimensional.

Completion Rates

The proportion of students completing the Basic Skills Courses were computed for both the laboratory and control groups. It was hypothesized that the completion rate would be higher for the laboratory group than for the control group. Although the laboratory group completion rate was higher, the small sample sizes did not allow the binomial distribution to be approximated by a normal distribution. Under the (questionable) assumption that both binomial distributions are normal, the experimental hypothesis was supported. ($\alpha = 0.05$, $z = 1.66$). Results are summarized in Table 15, below.

Table 15: Summary of Completion Rates

	Dual-purpose Group	Control Group
Number completing	26	23
Number starting	27	28
Proportion	0.96	0.82

The hypothesis that the Dual-purpose Group would have a higher percentage of students pass the course than the Control Basic Skills Group was supported, but not conclusively due to the size of the samples. It is likely that the additional contact with the same instructor, and the higher stake (three core requirement credits as well as Academic Skills Requirements) did effect improvement in persistence for the Dual-purpose Group. However, additional experimentation is required to establish this conclusion.

The proportion of students completing the Elements Course was computed for both the Dual-purpose Group and for the Elements Groups (taken as a single sample). It was hypothesized that there would be no difference in completion rate for the two groups. The hypothesis was supported. ($\alpha = 0.05$, $z = 0.27$, Critical $z = 1.96$). Elements pass rates are summarized on Table 16, below.

Table 16: Summary of Elements Pass Rates

	Dual-purpose Group	Elements Group
Number Passing	25	64
Number Enrolled	27	68
Proportion	.926	0.941

The Dual-purpose treatment was effective in providing remediation concurrent with a high pass rate for these students in their Elements course.

Related Results

Although not included in the experiment, the results on the pilot for the Elements achievement test for a homogeneous elements group taught by a colleague of the investigator resulted in test scores with mean and standard deviation results very similiar to those of the heterogeneous group. Results are summarized in Table 17.

Table 17: Summary of Means for Elements Test

Group	Sample Size	Mean Score	Standard Dev.
Pilot Group	45	16.2	5.6
Dual-purpose	24	15.33	3.95
Heterogeneous	33	15.91	4.49
Homogeneous	28	19.39	4.77

It was hoped that the dual-purpose treatment would be economical in terms of staff contact. This did not turn out to be the case, as the small size of the laboratory sections required more sections, compensating for the fewer contact hours per section.

Summary

The data supported two of the four hypotheses. The Dual-purpose Group mean pretest to posttest gain in arithmetic skills exceeded that of the Basic Skills Control Group. Also, the Dual-purpose Group pass rate in basic skills lab exceeded that of the the pass rate of the Basic Skills Control Group in the basic skills arithmetic course; and the Dual-purpose Group pass rate in the Elements course was not significantly lower than that of the other two Elements Groups. The dual-purpose treatment was successful in increasing achievement in arithmetic skills and in qualifying the dual-purpose students for the Elements course.

The data failed to support the hypothesis concerning the difference in achievement in the Elements Achievement Test. The mean for the Homogeneous Group was significantly higher than the mean for the Heterogeneous Group. The mean for the Heterogeneous Group was not different from that of the Dual-purpose Group. Also, the mean scores on the attitude scale did not differ significantly among the four groups. The dual-purpose treatment was not successful in preventing an adverse effect on the nonremedial students due to the presence of the remedial students in the same section. Also, the dual-purpose treatment did not increase attitude improvement for the Dual-purpose Group over the Basic Skills Control Group members. However, the Dual-purpose students did not score significantly lower than those of the two Elements Groups on the attitude test.

CHAPTER V

CONCLUSIONS

Introduction

This study investigated a basic skills option designed to enhance achievement, attitude, and persistence. An experimental dual-purpose program was developed following the general outlines of the CUPM recommendation for "Basic Mathematics." This program consisted of a one-credit mathematics laboratory course given concurrently with the beginning college-level liberal arts course "Elements." The overall intent of the study was to examine the feasibility of parallel remediation. It was hypothesized that a group of basic skills students could be remediated, while successfully completing the Elements course, without negatively effecting the achievement of the nonremedial students in the same Elements section. It was thought that this dual treatment would be accompanied by improved attitudes towards mathematics. The dual purpose program would be considered an unqualified success if

1. It was at least as successful as a more typical program in Arithmetic Skills Remediation.

2. The presence of the dual-purpose students in the Elements sections had no adverse affect on the achievement of their nonremedial classmates.

3. Its members made satisfactory achievement in the Elements course objectives as indicated by an ability to pass the course in the same proportion as their classmates who did not require remediation.

4. The attitudes towards mathematics of the dual purpose students would be more positive than those of the control group, and as positive as their nonremedial peers.

Based on the results of this study, the dual-purpose program was partially successful. The results indicate superior improvement in Arithmetic skills and a very satisfactory pass rate for both parts of the dual-purpose program. However, the hoped for improvement in attitude did not occur. Moreover, the presence of the remedial students in the Elements course had a negative effect on the achievement of the nonremedial students. This chapter includes a discussion of the limitations of the study, an interpretation of the results, the implications for current theory and practice in mathematics education, and some suggestions for future research.

Limitations of the Study

The study was not intended to investigate the isolated effects of various learning systems, teaching styles, and mathematical content. The approach was holistic in the sense that a dual-purpose system was compared to each of two single-purpose systems. Although a learning theory model was used as a basis for the design of the two courses used in the study, the postulated principles were not studied as independent variables. Thus the results of this experiment cannot be used to support or reject the validity of any particular learning theoretic postulates.

For example, even though the program made use of advance organizers, the outcome casts little light on the effectiveness of advance organizers on mathematical achievement, since all the Elements groups used the same advance organizers. The effect of a greater basic skills improvement for the Dual-purpose Group over the Basic Skills Control Group could be a consequence of any or all of the different variables that resulted from the attendence of the dual-purpose group in two courses. These different variables included mathematical content, teaching system, instructor, emphasis, and time on task, as well as the use of advance organizers.

Furthermore, the failure of the Dual-purpose Group to indicate better attitudes towards mathematics than the Basic Skills Control Group does not indicate neccessarily a failure of the theory used to develop the Dual-purpose courses. For example, other variables, such as the greater amount of content required of the Dual-purpose

Group due to their participation in two courses, might well have decreased the attitudes of this group. Such questions cannot be answered through the present study.

Interpretations and Implications

As indicated earlier in Chapter IV, the significant difference in achievement observed between the Heterogeneous and Homogeneous Elements Groups may be attributed to three possible sources: (1) lower teacher expectations due to the presence of remedial students, (2) the content of material presented to the Heterogeneous Group was weakened as a result of classroom interaction at a lower level due to the presence of the remedial students, and (3) the Heterogeneous Group members knew that many of their fellow students were remedial students.

Although in this experiment, the investigator was on guard against either conveying lower expectations or watering down content, it is reasonable to expect that some difference in expectation could have occurred, as well as some variation in content. The instructor spent more time with the members of this group, and consequently may have developed a stronger personal relationship. The small class size and the cooperative goal structure within the laboratory course also enhanced this effect. One Elements classroom effect of this was the more frequent participation of the Dual-purpose Group members in the class discussion. This more frequent participation could affect both the amount of mathematical content covered, and the response of the nonremedial students in the same classroom, contributing to underachievement on the part of the nonremedial students.

The knowledge on the part of the nonremedial students that many of their classmates were in a remedial class may have led to a belief that the class material would be easy and require little effort.

Some implications for practice are as follows: (1) heterogeneous grouping should be used with caution in the college mathematics classroom. The teacher must convey both verbally and non-verbally exactly what the expectations are for the class achievement. He/she must take extreme care that class discussion and presentation do not get bogged down at the level of the least prepared students. Such low level discussion and skill work should be taken up in coordinated laboratories or through the use of tutorial services. An extra effort to engage and challenge the nonremedial student is required in sections containing both remedial and nonremedial students.

2) The higher mean gain on arithmetic skills for the Dual-purpose Group can be explained in terms of the overall effectiveness of the dual-purpose treatment both in remediation and in success rate in the elements course. Such a dual-purpose program is undoubtedly preferable for many skill-deficient students to a separate sequence of remedial courses followed by a course such as Elements. The higher mean gain on arithmetic skills and the very high completion rate in the Elements course for the dual purpose group indicate that the parallel remediation approach is promising, and should be studied further.

The failure of the dual-purpose treatment to produce a gain in attitude over the more typical remediation treatment should not be interpreted as an equivalence of effectiveness of the learning systems involved as discussed above under limitations of the study. In fact, if attitude change is to be the object of study, it would be neccessary to design a study to implement more carefully the Dienes model while controlling other variables such as content. It is the view of this investigator that the failure to change attitude was a result of a failure to change the behavior of the students. The present experiment left the quality of time and effort in the "play stage" to the initiative of the student. Pressures of time, conflicting demands, and lack of intrinsic motivation may, for many students, have led frequently to insufficient time in the "play stage." The self-paced course, on the other hand, did have the advantage of requiring the student to build his own structures through the written text without benefit of the instructor's lectures. This fact together with the self-reinforcing nature of the Keller Plan type system may have resulted in greater anxiety reduction for the Control Group students, with a corresponding improvement in attitude. It is likely that the Dual-purpose Group had greater anxiety buildup through the neccessity to succeed in two courses, resulting in no improvement in attitude. This would have been compensated for somewhat by the attention to meaning and purpose in the Elements course, leaving the two groups in approximately the same place overall with respect to mathematics attitude.

Suggestions for Additional Research

The most pressing need for follow up research involves the underachievement of the nonremedial students in the heterogeneous Elements section. The dual-purpose students should be taught by different instructors in their two courses. The Elements instructor should be the same person for both homogeneous and heterogeneous sections as before. The Elements instructor should not be informed which section contained the dual-purpose proup members. The Elements instructor should take care to refer all "remedial type" questions to the learning center and see that tutorial help was available. They should be aware of the level of content and insure that the better qualified students do not become bored with the presentations or discussion. The Elements instructor should work hard to insure that expectations were clear, and are not conditional. With these additional controls, it may be possible to eliminate the under-achievement effect without harm to the remedial student. Another possibility is a separate section of Elements for the dual-purpose students.

The potential benefit for many remedial students in the dual-purpose program, as well as its efficiency in comparison to separate remediation courses, makes it advisable to continue working to develop a system within which it can be used with success for all involved. For example, a two-credit laboratory in basic arithmetic and algebra skills for the student marginally weak in those areas could be used in a dual purpose program. The parallel Elements course would be upgraded to include some applications of algebra along with those of arithmetic used in the present Elements course. Such a program will allow the highly motivated student, or the student returning after a break in his/her education, to get an immediate start into mathematics core and service coursework rather than postpone it for two semesters of remedial work. Of course, the student who requires pre-calculus or calculus study will be better advised to select a sequence of courses such as college algebra followed by pre-calculus followed by calculus.

Additional experimentation is needed to confirm or reject the conjectures about mathematics attitude and anxiety raised above. A measure should be devised to differentiate between such attitude questions as the importance of mathematics and mathematics anxiety. Further study with the Aiken Scale (1971) may show that it is such a measure.

Summary

This study investigated a basic skills option designed to enhance achievement, attitude, and persistence. An experimental dual-purpose program was developed following the general outlines of the CUPM recommendation for "Basic Mathematics." This program consisted of a one-credit mathematics laboratory course given concurrently with the beginning college-level liberal arts course "Elements." The overall intent of the study was to examine the feasibility of parallel remediation. It was hypothesized that a group of basic skills students could be remediated, while successfully completing the Elements course, without negatively effecting the achievement of the nonremedial students in the same Elements section. It was thought that this dual treatment would be accompanied by improved attitudes towards mathematics. The results of this study indicate that the dual-purpose program was partially successful. The results indicate superior improvement in arithmetic skills and a very satisfactory pass rate for both parts of the dual-purpose program. However, the hoped for improvement in attitude did not occur. Moreover, the presence of the remedial students in the Elements course appeared to have a negative effect on the achievement of the nonremedial students. Further studies are needed to explore ways of neutralizing this underachievement effect in the heterogeneous groupings. Additional dual-purpose programs should be developed to accelerate the entry of marginal skills-deficient students back into the educational mainstream.

APPENDICES

APPENDIX A

Mathematics Laboratory Course Material

MAT 097 LAB
Course Policy Guide

I. Purpose

The purpose of this course is to develop basic skills in arithmetic and your confidence in **your own ability to do mathematics.** We will also deal with attitude and anxiety issues involving mathematics. This course is coordinated with MAT 105D and 105F. You must be enrolled in MAT 105D or 105F now.

Skill areas that we will emphasize are the following:

1. Operations of addition, subtraction, multiplication, division.
2. Fractions.
3. Decimals.
4. Percents and ratios.
5. Problem solving.

At the conclusion of this course, you will take the arithmetic skills posttest.

If time permits, we will work on algebra skills, and you will have the opportunity to test out of a remedial algebra course.

II. Procedures:

Classtime will be used for demonstrations or class activities introducing the topic. Handouts will be given out to be worked on during the week. Most weeks we will meet only once. On weeks when a test is scheduled, we will meet twice. Retests will be given until a mastery level of 80% is reached. All tests must be passed before the final post-test is attempted.

Upperclass tutors will be available to assist.

III. Grading:

The grade for the course will be P or NP. A grade of NP requires the student to retake the course or take MAT 001. In order to pass the course, it is neccessary to pass achievement tests on each of the first four skill areas listed above and the final posttest on arithmetic skills.

It is expected that you will:

1. Attend all required classes.
2. Do all assigned worksheets.

3. Take required tests as scheduled.

IV. Other Information:

Tutors will be available in class and outside as arranged. See your instructor if you want tutorial help outside of class.

Office hours: MWF 10:00 - 11:30
TH 8:30 - 9:20

V. Course Syllabus

1. Unit 1: Review of whole number arithmetic.
   1. Box products.
   2. Distributivity
   3. Negative numbers
   4. Primes and LCM's
   5. Subtraction and division.

2. Unit 2: Fractions
   1. Definition
   2. Multiplication and division
   3. Addition and subtraction

3. Unit 3:Decimal fractions
   1. Examples
   2. The algorithms
   3. Ratios and percent

4. Unit 4: Algebra Operations
   1. Combining algebraic expressions
   2. Laws of exponents
   3. Polynomials
   4. Factoring

VI. Course Materials: The handouts for this course are from the Mathematics Text, Mathematics, An Exploratory Approach, by Sherman Stein, McGraw-Hill Book Company (Out of Print). The copies were made with the kind permission of Robert G. Stein, who informs me that he is in the process of revamping the book for a new edition. Chapters 1-7 and 8 are used as a laboratory manual for exercise in reading, problem-solving, and basic skills development.

Basic skills tests are similiar to the chapter tests of the text used in MAT 001 (Arithmetic Skills). The text for that course is Arithmetic: A Text/Workbook by Miller and Salzman published by Scott, Foresman and Company. Copies of this book are on reserve in the library. The arithmetic skills tests are based on the first seven chapters of Miller and Salzman.

APPENDIX B

Elements Course Material

MAT 105 Elements: Course Policy Guide   Frank Morgan

I. Assumptions

1. In the world around us, there exist form and structure, both natural and man-made.
2. Humans are gifted with an ability to observe, compare, operate, measure, understand and make predictions about this structure. This capability results in knowledge.
3. This knowledge can be shared and checked out with others.
4. Human knowledge is not comprehensive or complete, so we must continue to be open and learn. This includes me as teacher.
5. With knowledge comes a responsibility to care for the world. This is an individual and group endeavor. We can best fulfill this responsibility by working together while keeping individual accountability. How we do our work is as important as the final outcome.
6. Mathematics is a collection of disciplines which contributes to an understanding of the world and enables us to make predictions about process in the world.
7. "Mathematics can become a part of **every person's** understanding:

    a. A person deserves to have confidence in his or her ability to understand and think about mathematics.

    b. It is not a person's fault when that confidence is lost and it is always possible to regain it.

    c. Learning and thinking about mathematics need not be done in isolation. Collaboration increases achievement." (Rosamond, 1981).

    d. Mathematics is not a spectator sport. You have to get into it and "get your hands dirty."

8. You are here to learn mathematics. It is part of your job for the next 15 weeks. You will give it your best effort, and receive some satisfaction for your labor.

II. Purpose:

My intentions for this course are that you, the student, will have an increased appreciation of mathematics, an increased confidence in your own capability to do mathematics, better problem solving and thinking skills, and a mastery of the content of elementary mathematics.

III. Class procedures and grading:

1. I believe that mathematics learning requires you to build ideas about the structure of systems in the world. No one else can do this for you. Thus you are expected to prepare for every class by doing various reading and problem assignments. These assignments may be done in collaboration with others. Some of these will be turned in, and some will be used in class. In this class we will play with ideas and problems in much the same way that a child plays with blocks. We look for patterns and see how things are alike or differ, and how parts may be put together to make new objects. Classtime will be used for explanation, questions, discussion and pair or small group work. Like any job, this one requires your presence. I shall take attendence and expect an explanation for any absence or tardiness, preferably in advance. 15% of your grade will be based on classwork, attendance, and participation. More than two unexcused absences will result in a grade reduction.

2. A study sheet will be given each week with objectives and sample exercises. These sheets will be the basis of the weekly quiz which will be given later that week. 20% of your grade will be based on these quizes. Some of these will be takehome assignments.

3. After each unit we will have an achievement test. These are for learning, and you may retake a unit test to achieve a score of 80% correct or higher. Unit tests will account for 30% of your grade.

4. A final examination given at the end of the course will be the basis for 20% of your grade.

5. A semester project is required. This could involve the study of an application of mathematics to your major field, or a historical mathematical development, or a non-trivial problem that you wish to solve, or a mathematical system not dealt with in the course. You will select a topic by the end of the third week. A preliminary report of the project will be submitted at the end of the sixth week. The project is due by the end of the tenth week. 15% of your grade will be based on the project. Especially creative or excellent projects will receive extra credit.

Summary of grading basis:
1. Class attendence and participation..............15%
2. Weekly quizes.....................................20%
3. Unit tests..30%
4. Final Examination.................................20%
5. Semester project..................................15%

III. <u>Other Information:</u>
1. Tutors will be available.
2. Office hours: .... MWF 10:00 - 11:50 AM
T/TH 11:00 - 12:15
3. Phone: Office: 468-5611 ext 308
Home: 438-5590

IV. Course Syllabus

In this course we introduce two ways of viewing mathematics:

1. Mathematics is an active process or way to study the world for the purposes of understanding and prediction. We will define this process and illustrate it frequently in the course. This approach will be called "mathematical modelling."

2. Mathematics is the study of formal systems involving a set of objects, operations involving the objects, relationships among these objects, and rules which govern the behavior of the objects with respect to the operations and relations. We will also define this approach to mathematics and use it frequently in the course. This way of viewing mathematics will be called the mathematical systems approach.

LECTURE	TEXT READING
Unit 1: What is Mathematics?	
1. Introduction	
2. Patterns	p. 2-10
3. Problem solving	11-29
4. Mathematical proof	31-35
5. Set Theory	42-58
6. Cardinal number	61-72
7. Functions and relations	72-79
8. Logic and truth tables	80-91
9. Deductive reasoning	93-96
10. Test 1	
Unit 2: Number Systems	
11. The whole number system	101-149
12. Number theory	168-186

13. Integers	254-265	
14. Rational numbers	267-274	(213-248 assumed)
15. Equations and inequalities	277-281	
16. Problems	282-285	
17. Real numbers	286-292	
18. Review		
19. Test 2		

Unit III. Applications and modeling

20. Models for measurement	-
21. Non-decimal bases	151-161
22. Modular arithmetic	-
23. Finite systems	199-206
23. Distributions and Histograms	582-593
24. Descriptive Statistics	595-605
25. Test	
26. Graphs	469-479
27. Lines and other functions	480-489
28. Systems of equations	490-497
27. Equilibrium problems	-
28. Review and Summary	
29. Final Examination	

Unit 1: What is Mathematics?
Study sheet 1

The purpose of this unit is to introduce the broad themes that will guide us throughout the course. These themes include

1. Mathematics is the study of structure and pattern.
2. Mathematics is a tool in solving problems.
3. Mathematics is a language.

Mathematics learning involves becoming aware of the structure and pattern, capability with the operations and relations among the objects within the structures, and the ability to use correctly the conventional symbols, technical terms, formulas, and algorithms to express ideas and solve problems in the real world.

Part I: My claim is that just about any object or set of objects that you observe will have some mathematical component. For the purpose of the questions below, think of a structure as any object or set of inter-related objects.

Assignment: Write an answer to each of the following questions. Be prepared to share your ideas at the next class.

Study questions:

1. Give three examples of structure in the world around you, natural or man-made.

2. Describe any patterns that you can observe for each of these structures.

3. Can you imagine any problem involving one of the structures you described above? Describe the problem.

4. How would information about the patterns and structure help in the solution to the problem you described?

5. Did you use any mathematical language in answering questions 1-4 above? If so, was that language neccessary, or could you have done as well without it?

6. Describe any processes that you can think of that would help you learn more about the structures described in your answer to question 1.

7. Describe in your own words the idea of "private code"

presented in class. Does the use of special codes in mathematics have any bearing on math anxiety? Explain.

Part 2: Read the text, pp.2-10 and pp. 508-515.
<u>Objectives</u>:
1. Given a sequence of letters or numbers, find a pattern, describe the pattern verbally and by formula (if possible) and give the three next numbers in the pattern. Be able to define and illustrate <u>arithmetic</u> and <u>geometric</u> progressions. Work exercises 9/#3,5,7,15,26.

2. Given a formula for a pattern, find the first few terms in the resulting sequence. Work exercises p.9/#14.

3. Write an algorithm for some process that you do frequently. Express your algorithm as a flow chart. Use a given algorithm or flow chart to answer a particular question. Work exercises p.513/4,5,6,7.

Unit 1: What is Mathematics?
Study sheet 2

Week of 9/7

Part I. Read the text, pp.11-29.

1. In your own words, discuss the relationship of reading to mathematics. Do exercises p. 16/1,2,8,12,15,17.

2. Describe how Polya's problem solving process might be helpful in solving problems. Use Polya's method to solve at least two problems from p.30/5-26.

Part II. Read the text, pp.31-35.

1. Explain the difference between inductive and deductive reasoning. Which is most important in mathematics? Explain from your own experience. Prove or disprove a given statement. Choose two from among the exercises p.35/1,3,7,9

Unit I : What is Mathematics? Study Sheet 3 Week of 9/14

This week we will stress the "mathematical systems" approach by introducing one of the most fundamental systems, known as Set Theory. The homework assignments will serve to get you into the ideas and examples, while classtime will serve to organize and codify the material.

Part 1. Set Theory. Read the text, pp. 42-60. Make use of the worked out examples.

Be able to:

Give examples and work problems involving set relations and operations such as subset, proper subset, equal, complement, intersection, disjoint, union, ordered pair, Cartesian product.

You have to know the meaning of the terms. Work the following problems: 49/5,6,9,10,11,13,21 59/1,2,3,4,6,7,13,15.

Part 2: Cardinal Number. Read the text, pp. 61-71. Study the worked out examples and note new terms and symbols.

Be able to:

Give examples and work problems involving the following: one-to-one correspondence, equivalent sets,and cardinal number. Use Venn diagrams to solve counting problems. Work the following problems in the text: 70/1,2,3,4,5,6,7,14,17.

Mat 105 Study Sheet 4 Week of 9/21

Part I. In the modeling process, we are interested in finding a formula, graph, table of values, or some other rule which shows how two variable quantities are related. One of these is selected as the independent variable, so called because we can observe how changing its value affects the value of the other, called the dependent variable.

If this process is unique in the sense that each meaningful value of the independent variable corresponds to one and only one value of the dependent variable, we call it a function, otherwise we call it a relation.

Our textbook uses set theory to define relations and functions precisely as sets of ordered pairs. However, in everyday use, these ideas find expression in formulas, graphs, or tables.

Assignment: Search your textbooks (other than math) and find a good example of a relation or function in the form of a formula, table, or graph. Report your findings neatly on a single sheet. Include an explanation. Be sure to include the reference: Author, title, publisher, date, page. Natural and social sciences as well as business texts should provide lots of possibilities, but I am especially interested in the use of mathematical models in art, music, language, and any other less likely source. In fact, I will give extra credit throughout the semester for "new ones," so please keep your eyes open. This is an ongoing assignment.

2. Read pp.73-79 in the text, noting all definitions, examples and illustrations. Become familiar with textbook definitions of relation, function, reflexive, symmetric and transitive properties of relations, and equivalence relations. Note that we are still working in a "set theory" system.

3. Do exercises 79/ 2(a,b,c),3,5,6,7,13,14.

Part II. Proof in mathematics relies heavily on formal rules of logic. In this lesson we develop logic as a mathematical system, ending up with theorems that give some rules for deductive reasoning. The assignment is long, so be prepared to put several hours into it, otherwise you may not be able to keep up in class.

Assignment: (not to be handed in)

1. Read the text, pp. 80-94. Study the definitions and examples. Become familiar with the following terms: statement, negation, conjunction, disjunction, conditional, antecedent, consequence, inverse, converse, contrapositive, biconditional, tautology, rule of detachment, rule of syllogism.

2. Do the following:
86/2,5,8,9; 92/1; 94/1,2,4.

Unit 2 The Real Number System
SS5: The Whole Numbers
Week of 10/5

Part 1. We will treat the whole numbers as a <u>mathematical system</u>. This material is familiar to you all, at least in the sense of being able to represent quantities and perform calculations. However, you should read the text to "get the rust off", and to be sure you have an understanding of the common code which is needed to discuss the system properties of each number system as it is developed. These properties must be understood and be operational before algebra can be mastered.

<u>Assignment</u>:

1. Skim the text, pp. 100-150. Re-read any parts that seem unfamiliar. Write down the definitions of terms that appear in the reading. These should include the following:

binary operation, closure, commutative, associative, additive identity, multiplicative identity, distributive, algorithm, exponent, base, power, expanded form of a number.

Be able to use sets to illustrate the definitions of addition and multiplication.

2. Work the following problems:
106/106/1,2,3,8,9,11,15,16 116/1,2,6,7,10,17.
124/1,2,3,6,10,11 130/1,2,3,4,5
138/2,3,5,16,17
149/1(usual format),2,3(usual format)

<u>Do not hand in, but do these in your notebook and be prepared to ask or answer questions in class.</u>

Mat 105 Study Sheet 6 Week of 10/12

Part 1: In this lesson we will discover the concept of divisibility and its properties.

<u>Assignment</u>:

1. Read the text pp.168-188. Study the definitions and examples. Become familiar with the divisibility criteria, primes, composites, fundamental theorem of arithmetic, greatest common divisor, least common multiple, and applications including the Euclidean algorithm.

2. Work the following exercises: 173/2,7,9,10,16. 179/1,2,4,6,8,10,14. 187/1,3,5,6,11,12,14

Part 2: Integers

<u>Assignment</u>:

1. Read pp. 253-266. Be familiar by name with the system properties of the integers (p. 256). Study the worked out examples.

2. Work the following exercises:
261/1,5,7,8,9,10,13,16,17
265/1,2,3,4,6,7.

<u>Note</u>: Quiz week of 10/12 on ss5

Quiz week of 10/19 on ss6

Mat 105 Elements Week of 10/19 | Study Sheet 7 | Rational Numbers

Part 1: We review the operations and relations for fractions.

<u>Assignment</u>:

1. Skim the text, chapter 5, pp. 212-251. Study the parts that need the most review or seem most difficult. Especially notice all worked out examples that involve material or methods that you need to review carefully. Terms to know include: numerator, denominator, simplest form, equivalent fractions, improper fraction, reciprocal, terminating decimal, repeating decimal.

2. be able to do exercises like the following:
218/2,3,7,15,16,18. 225/1,2,4,5,7,12.
234/2,3,5,6,7,14
242/3,5,7,9. 248/1,3,6,7,13.

Part 2. We treat the rational numbers as a mathematical system that is built up from the set of integers by admitting all quotients (except division by zero).

<u>Assignment:</u>

1. Read the text, pp.267-274. Terms to know: rational number, multiplication inverse (reciprocal) property, field properties, trichotomy property, density property.

2. Be able to work exercises like the following:
274/1,2(a,c,e),4,5,6,7,10,11,12,13

Mat 105 Elements Study Sheet 8 The Real Numbers

Part I. We show the existence of numbers that are not rational, i.e. irrational numbers. We complete our number system study with the real number system.

Assignment:

1. Read the text, pages 286-292. Important terms include: irrational number, real number, completeness property. Know how to work with radicals and rational exponents.

2. Work the following exercises: 292/1,3,4,9,10,11,12,17,18.

Part 2: Test 2 Review.

Assignment:

1. Review the quizzes on whole numbers and integers.
2. Review ss5 - ss8. Among other things, know the system properties of whole numbers, integers, rational numbers, and real numbers, and how the systems differ with respect to properties. Be able to classify a number as whole, integer, rational, or real. Be able to work with rational exponents and radicals.

The test will consist of:
Closed book and notes, multiple choice questions (50%).
Open book and notes, problems (50%).

Mat 105 Elements SS9 Non-decimal Arithmetic

Part 1: Although our numeration system, the Hindu-Arabic system, uses powers of ten for place value, it is useful also to study the arithmetic of other bases. This will enhance your appreciation of the base ten system, and will make you aware of the system used in digital computers.

Assignment;

1. Read pages 151-156. Be able to
   i. Count in other bases.
   ii. Change a base ten numeral to another base.
   iii. Change from another base to base ten.

2. Do at home: 156/1,4,5,6,7,8,10,13,14,16,17,18,20.

Part 2:

Assignment:

1. Read pages 157-161. Be able to
   i. Compute the entries in the addition and multiplication tables for a given base.
   ii. Add, subtract, multiply, and divide in another base using numerals with more than one digit.

2. Exercises:
   162/1-8,10,11,12. Do "Just for fun" on p. 161.

Weekly Quiz: Part 1 objectives.

Mat 105 Elements SS10

Part 1: We have worked briefly (quiz 3) with finite systems. Now we will go into finite systems in more detail.

Assignment:

1. Read pages 199-206. Be able to

    1. Add, subtract, multiply, and divide using four minute clock and 12 hour clock.

    2. Form addition table and multiplication table for modulo n arithmetic. Add, subtract, multiply, and divide in modulo n.

Part 2. Continue part 1. Be able to

1. Given two numbers that are congruent modulo n, find all n for which the statement is true. (207/9(c,d))

2. Apply modular arithmetic in word problems. (207/3,6,7,8,15)

3. From the table of a finite system, decide whether the system is a group.

Mat 105 Elements SS11 Descriptive Statistics

Part 1: Graphical methods to describe data.

In this lesson we will learn how to set up a grouped frequency distribution, and then represent the data with graphs.

Assignment: Read pp.581-593. Terms to know: range, frequency distribution, class boundries, histogram, circle graph (pie chart).

Exercises: 587/1,2,3,7 593/1,3,4,5,7,9,11

Part 2: Numerical methods to describe data.

In this lesson we will learn the most common and important procedures for describing data sets numerically.

Assignment: Read pp.596-605. Terms to know: mean, median, mode, variance, standard deviation.

Exercises: 599/1(a,b,c),2, 3,4,10,15.
606/1(a,c,f),2,4,9,11,14

Mat 105 SS12 Weeks of 11/23 & 11/30

Part I. Semester Project work time. Your assignment for the week of 11/23 is for research, reflection, and making a first draft of your project. I will be available to examine your draft during the week of 11/30 for suggestions or encouragement (optional). <u>Your project will be due by 1 PM December 11.</u>
Office hours for week of 11/23:
Monday: 8:30 AM - 12:30 PM
Tuesday: 9:00 AM - 10:45 AM, 12:30 - 1:30 PM

Part II. Descriptive Statistics: Percentiles.

Percentiles are used to describe a person's relative performance on standardized norm-referenced tests. This lesson introduces percentiles and their relationship to frequency distributiona and histograms. We will use any extra time to review the objectives of the unit.

<u>Assignment</u>: Read pp. 608-612. Be able to find a given percentile for a set of scores or for grouped frequency distributions.

Exercises: 612/1,2,3,9,10.

Part III: Test 3: Other Bases, Modular Arithmetic & Statistics

<u>Type of test: Criterion referenced, multiple choice, one item for each of 30 objectives below.</u>

1. Objective: Given a picture of a set of multibase blocks, find the nondecimal numeral corresponding to the set (Comprehension).

2. Objective: Given a numeral in a base other than ten, find the numeral in that base for the next whole number (Comprehension).

3. Objective: write the base ten numeral for a number given in a another base (Application).

4. Objective: Change a given numeral to the base indicated (Application).

5. Objective: Given the digits of a number in base b and a representation of the same number in another base, find b (Analysis).

6. Objective: Compute an entry in the addition table for a given base (Application).

7. Objective: Compute an entry in the multiplication table for a given base (Application).

8-11. Objective: Add, subtract, multiply, and divide in another base using numerals with more than one digit (Application).

12. Objective: Perform addition using a twelve hour clock (Application).

13. Objective: Perform subtraction on a four minute clock (Application).

14. Objective: Given two numbers that are congruent mod n, find all n for which the statement is true (Synthesis).

15. Objective: Perform multiplication modulo n (application).

16. Objective: Perform division modulo n (Analysis).

17. Objective: Obtain a mathematical model using mod 7 arithmetic for finding the day of the week of a future event (Synthesis).

18. Objective: Define group in mathematics (Knowledge).

19. Objective: For a given finite set with a given binary operation, tell whether the system is a group. Construct an operation table if neccessary (Synthesis).

20. State the definitions of the measures of central tendency of a distribution of measurements (Knowledge).

21. Objective: Convert data given in the form of a grouped frequency distribution to a pie chart showing percentage in each class (Analysis).

22-23. Objective: For a data set, find the mean and standard deviation (Application).

24-25. Objective: predict the effect on the mean and standard deviation if data is changed systematically (Analysis).

26-30. Objectives: Interpret data given in histogram form, including percentile scores, mean, range, and the result of transforming to another form such as a pie chart (Analysis).

Mat 105 Elements SS13 Analytic Geometry

Part 1: We define the Cartesian coordinate system and show how it is used to provide visual models for algebraic formulas.

Assignment:

1. Read the text, pp.469-484. Number line, Cartesian Coordinate system, origin, coordinates, linear function, graph.

2. Exercises to do: 472/4(a,b,c), 5(a,b,c), 6(a,b,c).
479/1,4(a,b,c),5(a,b,c) 484/1,5,9,11,12

Part II: We define linear functions and show how these are used in modelling.

Assignment:

1. Read pp. 486-496. Become familiar with the following: slope, y-intercept, elimination method.

2. Exercises: 489/ 1,2,7,9,11 496/3.

Mat 105 Elements   SS14   Final Exam Items

Final Exam: 1 essay question, 17 applications, 2 analysis.

1. Unit 1: What is Mathematics? <u>Four questions</u>.
    1. How is a mathematical system built up? What are the ingredients of a mathematical system?
    2. Describe four steps in the mathematical modelling process.
    3. Given the first few terms in a sequence, find the next few. Tell if the sequence is arithmetic or geometric.
    4. Compute unions, intersections, complements, and cartesian products of given sets.
    5. Use Venn diagrams in classification problems.
    6. Construct a truth table for a given statement form and decide whether it is a tautology.

2. Number Systems. <u>Two questions.</u>
    1. Find greatest common divisors and least common multiples of a given number.
    2. Convert from fractional form to decimal and from decimal to fraction, including the case for non-terminating decimals.

3. Other bases, modular arithmetic, descriptive statistics.
    <u>Ten questions</u>.
    1. Count in other bases.
    2. Convert from one base to another.
    3. Add, subtract, multiply and divide in other bases.
    4. Add, subtract, multiply, and divide modulo n.
    5. Given a set of measurements, find
        a. Frequency histogram.
        b. Mean, median, mode.
        c. Range, variance, standard deviation, percentile score.

4. Analytic Geometry: <u>Four questions</u>.
    1. Sketch the graph of a given function.
    2. Given the formula for a linear function $y = mx+b$, find the slope, y-intercept, and sketch the graph.
    3. Solve a system of two linear equations in two unknowns using the elimination method.
    4. Use linear equations as models to solve problems.

APPENDIX C

Arithmetic Course Policy Guide

MAT 001
Essential Mathematics
Course Policy Guide

1. Course Objectives:

The text provides chapter objectives, explanations, examples, practice problems, and a practice test for each chapter. Your task is to master the objectives, and demonstrate your mastery by scoring at least 80% on a chapter test. When you have completed all chapters, you will take a Final Post-test. The textbook is Arithmetic: A Text/Workbook, by Miller and Salzman, and is on sale in the college bookstore. You are required to complete the first seven chapters of the text.

2. Procedures:

Class time will be used for study, 1-1 or small group instruction, or testing. You may work by yourself or quietly with a small group. Whenever you need help, go to a tutor. As soon as you are ready, take the chapter test. Your test will be corrected immediately by a tutor. You may be allowed to change an answer if you have made a trivial mistake and can find it on the spot. You will be given half credit if your changed answer is correct. The tutor will explain about any wrong answers, or may ask you to explain your work in some cases. If you score less than 80%, you will continue to work on the objectives that need further work. After studying these, you may take a retest, and so on, until you achieve a score of 80% or higher.

III. Attendance:

Attendance for the whole period of each class is required of each student. More than 3 cuts may result in a grade of NP for the course. Excused absences are to be authenticated in writing from the Dean, coach or other faculty member, or Doctor or nurse.

IV. Pacing:

Since this is a self-paced course, it is possible to finish early. Incentives for maintaining an adequate pace include:

1. Midsemester grades will be based on the number of units passed. 4 or more units completed = P
2. When you finish the required units, and pass the post-test, you may concentrate on other courses.

V. Staff:

1. Tutors: Tutors are available during class hours or other announced times to give 1-1 or small-group instruction. Their first priority during class time is to correct your test in your presence, to determine whether you have adequately mastered a given chapter. Tutors will be tough, but fair, in maintaining the criteria for passing tests.

2. Instructor: Your instructor will act as a tutor and proctor during class time. He is the final arbitrator of any questions that arise in tutoring or testing.

APPENDIX D

Elements Achievement Test Information

Achievement Test Construction: Elements

1. Purpose of the test

To measure whether there is any difference in achievement among the three groups -- Lab group, Experimental Elements group, Control Elements group -- on knowledge and skills developed in a freshman level liberal arts mathematics course.

2. Specifying content

Since the test is to measure knowledge and skills developed in the course, it was important to select material which was unlikely to be already known to the student. Since the Elements course is a survey, much of the content has been previously studied by some of the students. I selected the content that was least likely to have been studied in school mathematics. This content included:

1. Non-decimal numeration systems, including conversion and computation.
2. Modular arithmetic.
3. Descriptive statistics.

3. Type of test:

The test was designed to measure achievement rather than aptitude or creativity under pressure. A criterion referenced test based on behaviorial objectives best suited my purposes. I attempted to provide a reasonable distribution of cognitive skill levels using Bloom's taxonomy.

4. Steps in building test:
   i. Develop objectives.
   ii. Decide on cognitive skill level of the objective/item, using Bloom's taxonomy, balancing levels.
   iii. Develop table showing the above categories.
   iv. Decide on test format: multiple choice vs.

problems.

v. Write sample items - three for each objective.

vi. Review by experts, for technical problems as well as validity.

vii. Additional editing based on vi.

viii. Set standards of performance and scoring criteria.

ix. Assemble the test and directions.

x. Pilot test administered.

xi. Additional editing as needed from viii, xi.

5. Administration considerations:

i. Be sure directions are clear.

ii. Standardization issues considered, e.g. physical conditions, time for work, scoresheets, proctoring, space.

iii. Scoring considerations.

6. Test analysis

i. Item analysis.

ii. Distribution study.

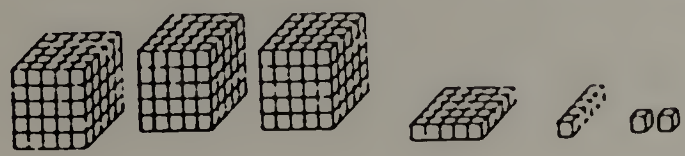
iii. Consistency by Kuder-Richardson 20.

MAT 105 Test 3

Name:.................................

Mark an X through the letter of the correct response on the answer sheet. You may do your scratch work on the test sheet.

1. The picture below shows a set of multibase blocks.



The numeral that best represents the set of blocks is

a. 3112_{four}

b. 3112_{five}

c. 3112_{six}

d. 3112_{ten}

2. The next whole number larger than 677_{eight} is

a. 678_{eight}

b. 670_{ten}

c. 680_{eight}

d. 700_{eight}

3. The base ten numeral for 2121_{four} is

a. 152_{ten}

b. 153_{ten}

c. 163_{ten}

d. 2121_{ten}

4. The base twelve numeral for 537_{ten} is

a. 389_{twelve}
b. $38T_{twelve}$
c. 399_{twelve}
d. 489_{twelve}

5. Given that $2341_{b} = 346_{ten}$, then b =

a. four

b. five

c. six

d. twelve

6. The sum of three plus seven in base eight is

a. ten

b. 10_{eight}
c. 12_{eight}
d. 13_{eight}

7. The product of five times seven in base eight is

a. Thirty five

b. 35_{eight}
c. 42_{eight}
d. 43_{eight}

8-11. For questions 8-11, use the base four tables below to select the correct base four answer for the given operations.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	10
2	2	3	10	11
3	3	10	11	12

X	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

8. $313_{four} + 112_{four} =$
   a. 431_{four}
   b. 1021_{four}
   c. 1030_{four}
   d. 1031_{four}

9. $312_{four} - 113_{four} =$
   a. 132_{four}
   b. 133_{four}
   c. 199_{four}
   d. 203_{four}

10. $231_{four} \times 13_{four} =$
   a. 3003_{ten}
   b. 4323_{four}
   c. 10323_{four}
   d. 10333_{four}

11. $321_{four} + 3_{four} =$

a. 103_{four}

b. 107_{four}

c. 110_{four}

d. 113_{four}

12. On a twelve hour clock, the sum of 8 and 7 is

a. 2

b. 3

c. 13_{twelve}

d. 15

13. On a four minute clock, the difference 1 - 3 =

a. Nonexistent.

b. 1

c. 2

d. 3

14. If 3 and 11 are congruent modulo n, than n <u>has</u> to be

a. 2.

b. either 2 or 4.

c. either 2 or 8.

d. either 2 or 4 or 8.

15. Compute 6 X 7 (mod 8). The answer is

a. 1

b. 2.

c. 3.

d. 5

16. Compute $3 \div 4 \pmod 5$. The answer is

a. non-existent.

b. 1

c. 2

d. 3

17. On Thanksgiving day (Thursday), we can say that there are 22 days left in the semester. We wish to find what day of the week that the semester ends. A mathematical model for this problem, using Sunday = 1, Monday = 2, etc. is

a. $5 + 22 \equiv n \pmod 7$

b. $n \equiv 22 \pmod 7$

c. $22 \equiv 5 \pmod 7$

d. $n + 5 \equiv 22 \pmod 7$

18. Which of the following is not neccessarily a property of a group?

a. Commutative
b. Closure
c. Associative
d. Identity

19. For the set $\{1,2,3,4\}$ under modulo 5 multiplication, a study of the operation table shows that

a. all group properties hold except the inverse.

b. the associative property fails.

c. the system is an Abelian group.

d. the system is a group but not Abelian.

20. The median score of a distribution is

a. The middle entry when scores are ranked from smallest to largest.

b. The score that occurs most frequently.

c. The average found by adding all scores and dividing by the number of scores.

d. The difference between the largest and smallest score.

21. Consider the following frequency distribution.

Résidence Status	Frequency
Ellis	105
Morrill	140
Haskell	145
Adams	148
Wheeler	138
Wright	19
Commuters	635

In a pie chart for this data, the percentage of area representing the number of commuters would be

a. about 14%.

b. slightly less than 50%.

c. more than 50%.

d. much larger than all the others put together.

22. For the data set $\{1,2,2,3,4,6\}$ the mean is

a. 2
b. 2.5
c. 3
d. 5

23. For the data set of #22, the standard deviation is

a. 1

b. 1.7

c. 2.7

d. 5

24. If 3 is added to each number in a data set, the resulting set will have standard deviation that

a. is the same as the original set.

b. is less than the original set.

c. is more than the original set.

d. may be more or less depending on the original data set.

25. If the mean of a set of measurements increases, the standard deviation

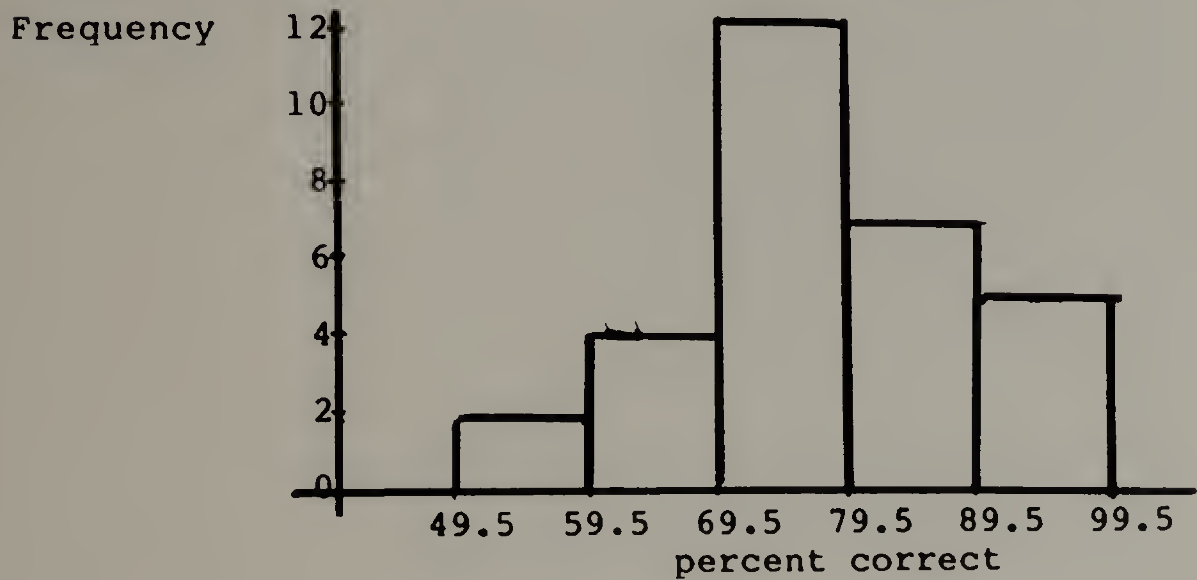
a. neccessarily decreases.

b. neccessarily stays the same.

c. neccessarily increases.

d. may stay the same or change.

26-30. The histogram below represents the scores on a statistics test:



26. The 85th percentile score is approximately

a. 80

b. 85

c. 90

d. 95

27. The mean score is probably about

a. 69

b. 77

c. 85

d. 89

28. If the data in the histogram were put on a pie chart, the portion of the circle representing those who scored 70 or higher is about

a. 60%

b. 70%

c. 80%

d. 90%

29. The range of the data set for the histogram is

a. More than 50.

b. Between 30 and 50.

c. Less than 30.

d. Not able to be bounded.

30. Bill scored 80 on the statistics test represented in the histogram. His percentile score was about

a. 40

b. 50

c. 60

d. 70

Table 18. Breakdown of Test Items in Bloom's Taxonomy

Item	Know	Comp	App	Ana	Syn	Eval
1		x				
2		x				
3			x			
4			x			
5				x		
6			x			
7			x			
8			x			
9			x			
10			x			
11			x			
12			x			
13			x			
14					x	
15				x		
16				x		
17					x	
18	x					
19					x	
20	x					
21				x		
22			x			
23			x			
24				x		
25				x		
26				x		
27				x		
28				x		
29				x		
30				x		
Totals	2	2	12	11	3	

APPENDIX E

Aiken Mathematics Attitude Scale

Directions: Write your name in the upper right corner. Then draw a circle around the letter indicating how strongly you agree or disagree with each statement. SD (Strongly Disagree), D (Disagree), U (Undecided), A (Agree), SA (Strongly Agree).

1. Mathematics is not a very interesting subject.	SD	D	U	A	SA
2. I want to develop my mathematical skills and study this subject more.	SD	D	U	A	SA
3. Mathematics is a very worthwhile and neccessary subject.	SD	D	U	A	SA
4. Mathematics makes me feel nervous and uncomfortable.	SD	D	U	A	SA
5. I have usually enjoyed studying mathematics in school.	SD	D	U	A	SA
6. I don't want to take any more mathematics than I have to.	SD	D	U	A	SA
7. Other subjects are more important to people than mathematics.	SD	D	U	A	SA
8. I am very calm when studying mathematics.	SD	D	U	A	SA
9. I have seldom liked studying mathematics.	SD	D	U	A	SA
10. I am interested in acquiring further knowledge of mathematics.	SD	D	U	A	SA
11. Mathematics helps to develop the mind and teaches a person to think.	SD	D	U	A	SA
12. Mathematics makes me feel uneasy and confused.	SD	D	U	A	SA
13. Mathematics is enjoyable and stimulating to me.	SD	D	U	A	SA

14. I am not willing to take more than the required amount of mathematics. SD D U A SA

15. Mathematics is not especially important in everyday life. SD D U A SA

16. Trying to understand mathematics doesn't make me nervous. SD D U A SA

17. Mathematics is dull and boring. SD D U A SA

18. I plan to take as much mathematics as I can during my education. SD D U A SA

19. Mathematics has contributed greatly to the advancement of civilization. SD D U A SA

20. Mathematics is one of my most dreaded subjects. SD D U A SA

21. I like trying to solve new problems in mathematics. SD D U A SA

22. I am not motivated to work very hard on mathematics lessons. SD D U A SA

23. Mathematics is one of the most important subjects to study. SD D U A SA

24. I don't get upset when trying to do mathematics lessons. SD D U A SA

APPENDIX F

Pilot Test Results

The Elements Achievement Test was conducted on November 4, 1988 at Castleton State College in Castleton Vermont. The results are summarized in Table 18 below.

Table 19: Pilot Test Results

Number of students tested	45
Number of test items	30
Mean score	16.2
Standard Deviation	5.25
r_{tt}	0.79

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